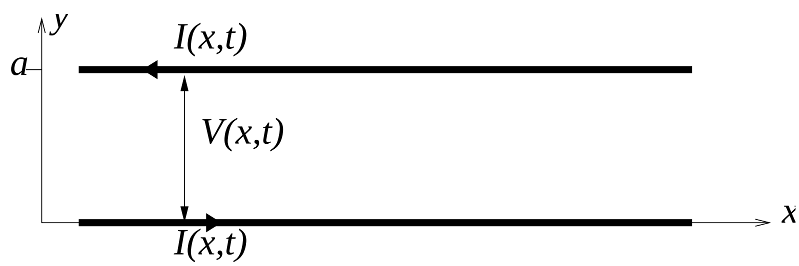
EMB: Consider an infinitely long transmission line consisting of two perfectly conducting wires running parallel to the x axis at $y = 0$, and $y = a > 0$ and separated by a dielectric. The wires have capacitance C per unit length and inductance L per unit length. In this problem the *longitudinal* components E_x and B_x both vanish everywhere, so we are examining *transverse electromagnetic* (TEM) waves.



a) Let $V(x,t)$ be the position- and time-dependent EMF between the wires that is defined by evaluating

$$V(x,t) = -\int_0^a E_y(x,y,t)dy$$

along a straight path between the wires at fixed x. Show that

$$\frac{\partial V}{\partial x} = \kappa \frac{\partial I}{\partial t}$$

for some constant κ depending on L and/or C that you should find. Here $I(x,t)$ is the current at x in the lower ($y = 0$) wire. The current at x in the upper ($y = a$) wire has the same magnitude, but opposite direction.

b) By using the charge continuity equation, derive a similar relation between $\partial V/\partial t$ and $\partial I/\partial x$.

c) Combine the differential equations from parts (a) and (b) to obtain a wave-equation for $I(x,t)$, and express the wave velocity c_{TEM} in terms of L and C.

d) If $I(x,t) = I_0 \sin(kx - c_{\text{TEM}}t)$ what is $V(x,t)$?

e) By what factor will the RMS power transmitted by the line change when L is increased by a factor of 10 while keeping I_0 fixed.