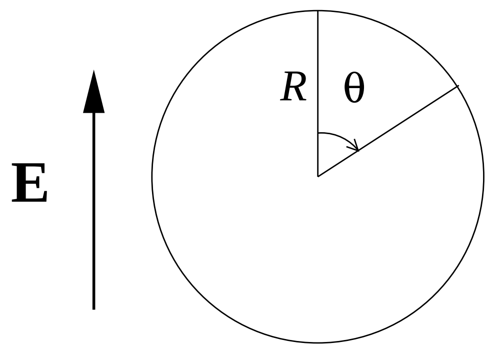
EMA: When a perfectly conducting sphere of radius R is placed in an initially uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$ it develops a surface charge distribution $\sigma = 3\epsilon_0 E_0 \cos\theta$ where θ is the spherical polar coordinate.



a) What is the total charge on the sphere?
b) Calculate the electric dipole moment $\mathbf{p}$ of the sphere.
c) If we take the potential of the surface of the sphere to be zero, what is the potential $\varphi(\mathbf{r})$ at all points $\mathbf{r}$ *outside* the sphere? Express your answer in terms of $\mathbf{p}$. (Remember to include the potential of the field $\mathbf{E} = E_0\hat{\mathbf{z}}$. As a check of your computation verify that your expression for $\varphi(\mathbf{r})$ is indeed constant everywhere on the sphere's surface.)

We now build an artificial dielectric consisting of identical perfectly conducting metal spheres of radius R which are randomly distributed in an unpolarizable medium. The average number of spheres per unit volume is N. Assume that when we insert the dielectric in a uniform field we can neglect the difference between the actual electric field acting on each sphere and the applied uniform electric field (*i.e.*, the polarizability is weak).

d) Starting from an appropriate Maxwell equation, derive the relation between the electric field $\mathbf{E}$, the electric displacement field $\mathbf{D}$, and the polarization $\mathbf{P}$ in a dielectric material.
e) Use your results from parts (b) and (d) to compute the dimensionless dielectric constant (also called the *relative permitivity*) of our artificial dielectric.