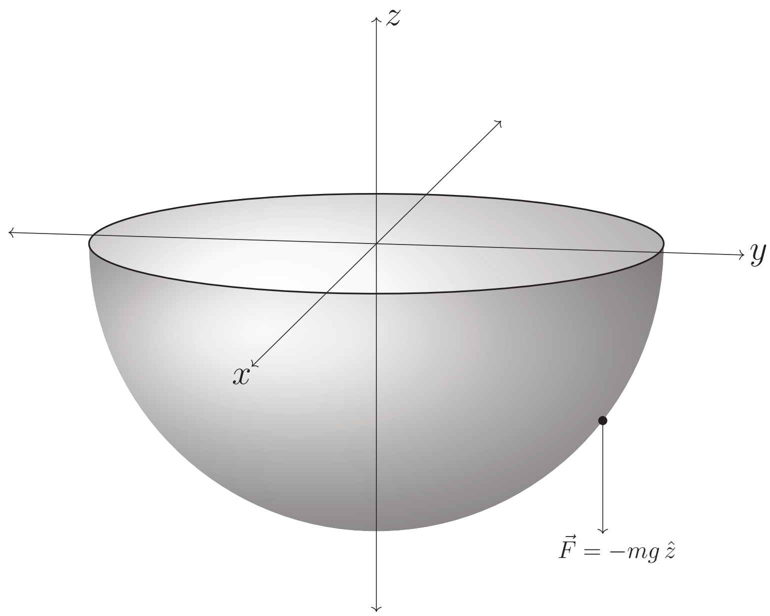
**CMB**: Consider a point particle of mass $m$ subject to a gravitational force $-mg\hat{\mathbf{z}}$ and constrained to move on the 2-dimensional frictionless surface of a lower hemisphere of radius $R$ centered at the origin.




a) Write down the Lagrangian for the problem, and from it find the Euler-Lagrange equations for the motion of the particle in $r, \theta, \phi$ spherical coordinates.

b) The particle is orbiting the hemisphere at a constant polar angle $\theta_0$ with respect to the $z$-axis, where $\pi/2 < \theta_0 < \pi$. At time $t = 0$, the particle is at $(r, \theta, \phi) = (R, \theta_0, 0)$ and its velocity is in the $+\hat{\phi}$ direction. Find the coordinates of the particle for $t > 0$.

c) Suppose the orbiting particle in part (b) is given an instantaneous impulse in the $-\hat{\theta}$ direction such that the resulting trajectory reaches the minimum polar angle $\theta_1$, where $\pi/2 < \theta_1 < \theta_0$. Find the equation that describes the maximum polar angle $\theta_2$ that the particle can take. Do not attempt to solve the equation for $\theta_2$.

d) In part (c), if you assume that $|\theta_1 - \theta_0| \ll 1$, then you can linearize one of the Euler-Lagrange equations as a harmonic oscillator equation for $\theta$ around $\theta_0$. Find the frequency of this oscillation.