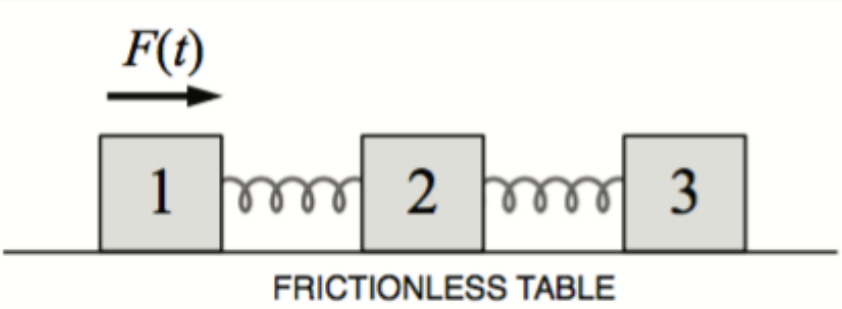
CMA: Three identical blocks of mass m slide on a frictionless horizontal surface and are constrained to move only along the $\hat{x}$ axis. Identical linear springs of strength $k \equiv \omega_0^2\, m$ connect the blocks.



At time $t = 0$, the blocks are at rest and the springs are at their unstretched lengths. Let $x_1,\ x_2$, and x_3 represent the deviations of the blocks' positions from their locations at $t = 0$. At times $t > 0$, an external force

$$F(t) = f\, m \cos \omega t$$

is applied to block #1. Here f and ω are constants. Your task is to figure out the motion $x_3(t)$ of block #3 for times $t > 0$.

The equations of motion of the blocks decouple when they are written in terms of the coordinates $\xi_1,\ \xi_2,\ \xi_3$, which are the *normal coordinates* of the system. Two of the normal coordinate are:

$$\xi_1 = x_1 + x_2 + x_3, \quad \text{and} \quad \xi_2 = x_1 - x_3.$$

a) Find the third normal coordinate, ξ_3,
b) Find the decoupled equations of motion of the system,
c) Find the position $x_3(t)$ of block #3 for times $t > 0$.