Proceedings of the 50th Hawaii International Conference on System Sciences | 2017

Optimal Electricity Pricing for Societal Infrastructure Systems

Mahnoosh Alizadeh
Stanford University

Hoi-To Wai
Arizona State University

Andrea Goldsmith
Stanford University

Anna Scaglione
Arizona State University

***Abstract*—We develop a general framework for pricing electricity in order to optimally manage the electricity load of societal infrastructures that interact with power systems through their price-responsive electricity load. In such infrastructure systems, electricity is not the sole resource needed to serve users' needs. Examples include cloud computing infrastructure or electric transportation networks. In these cases, other shared networked resources such as charging stations or communication links and data centers are also required to serve users. Hence, pricing of electricity becomes intertwined to managing other congestible resources not priced by the power system operator, leading to a complex economic dispatch problem. For brevity of notation, our analysis is performed under a static setting. We discuss how the power system operator should model the effects of the mobility of loads and congestion in the infrastructure in the economic dispatch. We numerically study the performance of our algorithms using the example of a simple electric transportation network.**

I. Introduction

Demand-side management programs aim to influence the amount and timing of our societal resource consumption to improve efficiency. In particular, dynamic pricing programs in power systems offer operators a chance to tap into the enormous opportunities that would follow from a more price-sensitive demand behavior. However, making the demand for resources flexible comes with its own set of challenges, mostly geared towards finding *the right price*. A significant body of literature has been dedicated to studying the different aspects of electricity pricing in the presence of flexible loads (see, e.g., [1]–[6]). An implied assumption in these works is that electricity is the sole congestible resource that electric loads need. However, our work is motivated by the fact that demand management introduces a novel form of coupling (feedback) between the grid and societal-scale networked infrastructures with flexible electricity load. The reason is that such infrastructures do not depend solely on electricity to serve end-users and have their own congestion management problems. We provide examples in the next paragraph.

A significant amount of flexibility could follow from the electricity demand that supports the delivery of goods and services by networked infrastructure systems. Prominent examples include: cloud computing services and Internet data centers [7]–[10] and electric transportation systems [11]–[14]. Unlike traditional responsive loads (such as HVACs), here electricity consumption is coupled with another form of constrained (congestible) network resource to serve users. Whether it's by routing the workload to capacity-constrained data centers through links with limited bandwidth, or by routing Electric Vehicle (EV) owners to charge at different locations considering the limited capacity of charging stations and feeders, network service retailers' electric load flexibility is tied to their ability to overcome congestion and manage mobility in their respective infrastructure system. Since such congestion also incurs costs for the retailers, their overall service decisions would not be solely based on their electricity costs. Hence, their load flexibility cannot be described using a simple bid to the wholesale power system operator. As shown in [16], ignoring this coupling of objectives between electricity cost minimization and network management can result in operational instabilities for the power grid. This interests us in studying electricity pricing in such cases.

We consider the problem of wholesale price design to tap into the load flexibility of network service retailers that serve users of societal infrastructure through long-term subscriptions. We assume that the retailers have direct control over the Quality of Service (QoS) provided to customers and need to control their costs while maintaining QoS. Each retailer owns (or co-owns) a number of geographically-dispersed service centers. Example of this includes EV drivers subscribing to a network of charging stations owned by a company and being directly assigned to optimal charging locations or tasks routed to and served via Internet data centers.

Prior art regarding electricity price design for price-responsive infrastructures: The authors in [14] consider the case where the operator tracks the mobility of large fleets of EVs and their energy consumption and designs optimal multi-period Vehicle-to-Grid strategies. In our previous work in [15], [16], we studied how non-profit transportation and power system operators can collaborate together to consider an interconnection introduced between their systems through EVs. Some of the main assumptions that [15], [16] rely on are that EV owners directly buy electricity from the system operator (i.e., no retailers), and that all transportation links and charging stations can have Pigovian taxes, e.g., tolls, imposed on them. Similar assumptions were adopted in [17], where the authors study the EV charge control problem with wireless charging. We remove these assumptions here by including retailers in the picture and studying the effects of removing tolls. We concur that our model does not yet capture all the intricacies of this complex problem, but we consider it a first step towards a more holistic treatment.

Outline: We first introduce the major players in this problem in Section II. This includes the wholesale power system operator and the network service retailers that serve infrastructure users (and buy electricity from the wholesale market to do so). We also define the concept of *virtual paths*

URI: http://hdl.handle.net/10125/41523
ISBN: 978-0-9981331-0-2

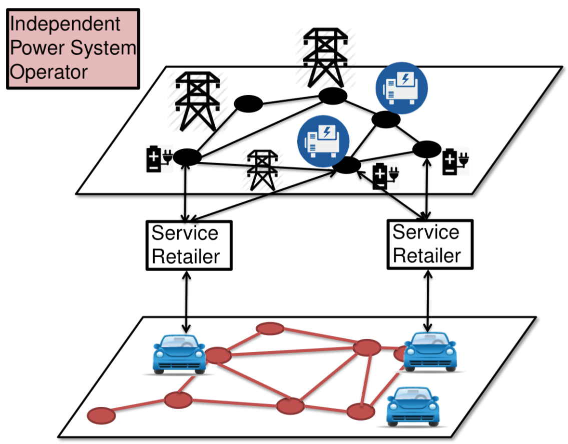

Fig. 1. The operations of the power system and an electric transportation network coupled together through service retailers (charging network operators). These charging network operators buy electricity to serve users and also manage congestion in the charging facilities due to limited number of stations and feeder constraints. The could potentially provide route guidance to electric vehicles owners as well.

on an extended graph which helps to model the effect of the retailers' decisions on electricity load. In Section III, we model each player's decision making process mathematically. In Section IV, we study whether we can enforce socially optimal consumption behavior through electricity pricing with the help of congestion tolls. In Section V, we study the scenario where the only way to influence the retailers' electricity consumption is through electricity prices, and no tolling is possible. Finally, Section VI is dedicated to a numerical study in electric transportation systems.

II. The Basics

Here we describe the basic elements in the picture when a societal infrastructure such as the electric transportation system has price-responsive load (see Fig. 1).

A. Infrastructure System

We consider a set of geographically-dispersed heterogeneous service centers $\mathcal{S}$ that use electric energy to serve customer needs and support the functionality of an infrastructure system. These service centers are connected via a directed connectivity graph $\mathcal{G}_I$. This connectivity graph models a network of congestible links that allow goods to be transported geographically. For example, in an electric transportation network, the arcs correspond to roads, and the service centers correspond to battery charging stations.

B. Power System

The power system provides electricity to service centers of the infrastructure system and is represented through a graph $\mathcal{G}_P = (\mathcal{V}, \mathcal{L})$. Each node $v \in \mathcal{V}$ on the power grid graph has an associated price for electricity p_v that is chosen by the Independent Power System Operator (IPSO). The pricing mechanism we choose in this paper is Locational Marginal Pricing (LMP) [18] (simple definition given in Section III-C). If electricity was the sole resource the retailers needed to serve users, the LMP mechanism would have induced a welfare-maximizing load and generation profile.

C. Service Retailers

We consider the scenario where the service centers that keep the infrastructure system functional are operated by competing retailers to serve their population of subscribers. Each end-user subscribes to one such retailer $r \in \mathcal{R}$. The subscriptions are long-term contracts that we consider as given in this work. The retailers need to assign incoming jobs to service centers by considering the effect of geographically-variant power system prices in their costs among other variables such as transport delays and service center capacities.

D. End-user Needs

Heterogeneous user needs originating at different nodes of the infrastructure network usually require one of the two following types of service:

1) Transport networks: in this type of network, the jobs specify the need to move people or material to different destination nodes. We assume that electric service centers are required to enable this transport. This could represent battery charging in electric transportation networks, or pumps in water networks.
2) Resource-sharing networks: in this type of network, transportation of goods is not the primary goal of the infrastructure. Rather, the users require resources that can be made available to them via one or more of the geographically-dispersed service centers. Hence, transportation is a secondary service to make shared resources available to users. An example is cloud infrastructure, where high speed transport of the data is a knob to enable data processing as a shared service and cloud-to-cloud interoperability. In short, here transportation enables resource sharing at service centers.

In both type of networks, each retailer needs to pick the best combination of routes on the connectivity graph and service delivered by the service centers it owns in order to best serve its users. The criterion applied for the optimum planning is to minimize operational costs while keeping the users happy about the quality of service (QoS) they receive.

E. Feasible Virtual Paths

Each retailer needs to capture the aggregate effect of choosing different transport and service options for each arriving job from its subscribers on the total incurred cost. While it is quite natural to think of the transportation side of the retailers' objective as a network flow assignment problem [19] (and the pertinent literature provides a clear understanding of how different path choices for transport will affect aggregate network flow), flow models are not readily adaptable to resource allocation choices at service centers. But service centers are exactly where electricity is consumed and what interests power system operators. Hence,

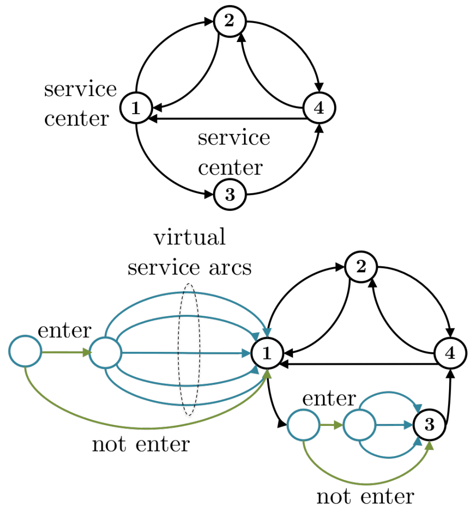

Fig. 2. Top: infrastructure system connectivity graph $\mathcal{G}_I$, with service centers at nodes 1 and 3; Bottom: extended graph $\mathcal{G}_I^e$ with virtual entrance and service arcs (representing finite number of service choices).

in capturing the effect of service decisions on electricity costs, the challenge is to find the right models to capture the aggregate effect of retailers' decisions to process arriving jobs at different service centers (and with different QoS levels) on the power load. As an example of this challenge, consider a retailer that needs to route a number of EV owners across congestible roads and charging stations while providing them with the right amount of battery charge so as to avoid the EV batteries being depleted mid-travel. The challenge for this retailer is how to guide the route and charge decisions of each vehicle in an optimal manner so they can reach their final destination, where optimality is typically characterized by electricity and traffic congestion costs.

By assuming that only a finite number of processing choices are available at each service center for incoming jobs, e.g., a finite number of choices for how much an EV can charge their battery at a charging station, here we recast the joint transport and service problem as a network flow problem on a new *extended multigraph* $\mathcal{G}_I^e(\mathcal{B}, \mathcal{E})$. The extended graph $\mathcal{G}_I^e$ is defined by associating service decisions made at each service center to flows on a set of new *virtual service arcs*. These arcs are to be added at all such nodes of the graph $\mathcal{G}_I$ where a service center is located. To enter a service center, a job has to be pass through a virtual entrance arc that can capture congestion at the service center (see arcs labeled 'enter' in Fig. 2). The virtual arcs labeled 'not enter' have no travel cost and take zero time to travel. Upon arrival at the service center, the type of service to be received is represented by the choice of virtual service arc.

A job can be served by retailer r if it is routed through a *virtual path* on $\mathcal{G}_I^e$ that is *feasible* for retailer r. We define feasibility as follows: in a transport network, a path is feasible if 1) all arcs on the path are public or are privately owned by retailer r; 2) the service choices represented by the virtual arcs can provide the energy needed for transport, e.g., if enough battery charge is received for an EV to complete a trip. In a resource-sharing network, a path is feasible if 1) all arcs on the path are public or are privately owned by retailer r; 2) the work performed at service centers satisfies the job specifications, e.g., if the processing requirements of a job are met by the cloud infrastructure.

We use this notion of extended graph to solve a class of problems pertaining pricing electricity in coupled infrastructures. Relying on this abstraction, next we model all agents involved mathematically.

III. System Model

In this section, we model the electric load flexibility of an infrastructure using a network flow problem (see, e.g., [19]) on the extended infrastructure graph defined in Section II-E.

A. Infrastructure Network Flow Model

For each retailer $r \in \mathcal{R}$, arriving jobs can belong to one of finite classes $c \in \mathcal{C}^r$. The average rate of arrival of class c jobs is denoted by u_c. This is considered as a given static parameter in the retailer's decision problem. These arriving jobs can be served by being assigned to one of the feasible virtual paths $k \in \mathcal{K}_c$ on the extended graph $\mathcal{G}_I^e$ (see Section II-E). The rate of transfer (flow) of class c jobs assigned to virtual path k is denoted by f_c^k and is a decision variable for the retailer. The vector of path flow decisions made for class c jobs is denoted in vector form as $\mathbf{f}_c = [f_c^k]_{k\in\mathcal{K}_c}$.

We require that all arriving jobs be served. This translates into the following flow conservation constraint:

$$\mathbf{1}^T\mathbf{f}_c = u_c, \quad \forall c \in \mathcal{C}^r, \forall r \in \mathcal{R}. \tag{III.1}$$

Given the path decisions of all incoming jobs, the flow of jobs served by retailer r through arc a on the extended graph is given by $\lambda_a^r = \sum_{c\in\mathcal{C}^r, k\in\mathcal{K}_c} \delta_a^k f_c^k$, where δ_a^k is an arc-path incidence indicator (1 if arc a is on path k and 0 otherwise). This is written in matrix form as:

$$\boldsymbol{\lambda}^r = \sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c, \tag{III.2}$$

where $\boldsymbol{\lambda}^r = [\lambda_a^r]_{a\in\mathcal{E}}$ denotes the vector of network flows due to retailer r on the extended graph $\mathcal{G}_I^e$. The matrix $\mathbf{A}_c$ is a $|\mathcal{E}| \times |\mathcal{K}_c|$ matrix such that $[\mathbf{A}_c]_{a,k} = \delta_a^k$. The total flow on the extended infrastructure graph is given by:

$$\boldsymbol{\lambda} = \sum_{r\in\mathcal{R}} \boldsymbol{\lambda}^r \tag{III.3}$$

The flow on the virtual arcs of the extended graph leads to an electricity load. Let us denote the set of all virtual arcs associated with service centers connected to node v of the power grid as $\mathcal{E}^v$ and the corresponding electric load due to service centers as a vector $\mathbf{d} = [d_v]_{v\in\mathcal{V}}$. Then, we can write:

$$\mathbf{d} = \mathbf{M}\boldsymbol{\lambda}, \tag{III.4}$$

where $\mathbf{M}$ is a $|\mathcal{B}| \times |\mathcal{E}|$ matrix given by:

$$[\mathbf{M}]_{v,a} = \begin{cases} e_a, & a \in \mathcal{E}^v, \\ 0, & \text{else}, \end{cases}$$

and e_a denotes the energy consumed when a job is routed through virtual arc a. This could represent the amount of charge to be delivered to an EV battery, or the power consumed by processors serving a job at a data center (including cooling costs). Alternatively, we can distinguish between the demand due to each retailer and write (III.4) as:

$$\mathbf{d} = \sum_{r\in\mathcal{R}} \mathbf{d}^r = \sum_{r\in\mathcal{R}} \mathbf{M}\boldsymbol{\lambda}^r. \tag{III.5}$$

B. Individual Retailer Problem

Each retailer aims to serve all incoming jobs while minimizing the congestion costs on transportation and service arcs plus electricity costs. The time spent on an arc is considered a function of the flow on the arc. This is true for both transportation arcs, e.g., communication network delays as a function of rate, as well as for virtual arcs, e.g., processing delay of jobs or waiting at the charging station. Hence, we denote the congestion cost of arc a on the extended graph as a function $s_a(\lambda_a)$ of the flow on the arc λ_a, and we use $\boldsymbol{s}(\boldsymbol{\lambda}) = [s_a(\lambda_a)]_{a\in\mathcal{E}}$. We assume that $s_a(\lambda_a)$ can be expressed as:

$$s_a(\lambda_a) = \theta_a(\lambda_a)^{o_a} + \beta_a, \tag{III.6}$$

commonly referred to as the Bureau of Public Roads (BPR) delay function. The parameters (θ_a, o_a, β_a) are given constants for each arc. For all virtual arcs except for entrance arcs, we assume that $\theta_a = 0$.

Given the network flow due to all other retailers besides retailer r, denoted as $\boldsymbol{\lambda}^{-r} := \sum_{r'\neq r}\boldsymbol{\lambda}^{r'}$, each retailer r would optimize its own flow to minimize costs. Given the vector of LMPs, denoted by $\mathbf{p}$, we define an auxiliary cost function for each arc a on the extended graph as:

$$\boldsymbol{h}(\tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r};\mathbf{p}) := \boldsymbol{s}(\tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r}) + \mathbf{M}^T\mathbf{p}. \tag{III.7}$$

We also define transportation tolls imposed on each arc of the transportation graph (including virtual arcs) as $\boldsymbol{\eta} = [\eta_a]_{a\in\mathcal{E}}$. Accordingly, the cost function of retailer r is:

$$J^r(\tilde{\boldsymbol{\lambda}}^r;\boldsymbol{\lambda}^{-r};\mathbf{p}) := (\tilde{\boldsymbol{\lambda}}^r)^T[\boldsymbol{h}(\tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r};\mathbf{p}) + \boldsymbol{\eta}]. \tag{III.8}$$

Hence, the cost minimization problem of retailer r can be formulated as:

$$\begin{aligned} \min_{\mathbf{f}_c, c\in\mathcal{C}^r} \quad & J^r(\tilde{\boldsymbol{\lambda}}^r;\boldsymbol{\lambda}^{-r};\mathbf{p}) \\ \text{s.t.} \quad & \mathbf{f}_c \geq \mathbf{0},\ \mathbf{1}^T\mathbf{f}_c = u_c,\ \forall\ c\in\mathcal{C}^r, \\ & \tilde{\boldsymbol{\lambda}}^r = \textstyle\sum_{c\in\mathcal{C}^r}\mathbf{A}_c\mathbf{f}_c. \end{aligned} \tag{III.9}$$

Let us also denote the feasible set of $\tilde{\boldsymbol{\lambda}}^r$'s in (III.9) as $\mathcal{F}^r := \{\tilde{\boldsymbol{\lambda}}^r : \tilde{\boldsymbol{\lambda}}^r = \sum_{c\in\mathcal{C}^r}\mathbf{A}_c\mathbf{f}_c,\ \mathbf{f}_c \geq \mathbf{0},\ \mathbf{1}^T\mathbf{f}_c = u_c,\ \forall\ c\in\mathcal{C}^r\}$. Notice that $\geq$ denotes element-wise inequality.

The retailer optimization problems are coupled through the system-wide network flow variable $\boldsymbol{\lambda} = \tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r}$. The implications of this coupling will be discussed in Section IV. We next discuss how the IPSO sets the LMP $\mathbf{p} = [p_v]_{v\in\mathcal{V}}$.

C. The IPSO's Economic Dispatch Problem

To serve the electricity demand of the infrastructure, a set of generators are located at different nodes of the power network. For brevity, let us assume that a single merged generator is located at each node of the grid. Assuming that the generation at each node is denoted by a vector $\mathbf{g} = [g_v]_{v\in\mathcal{V}}$ and the baseload[1] by a vector $\boldsymbol{\ell} = [\ell_v]_{v\in\mathcal{V}}$, there are two constraints that define a *feasible dispatch* $\mathbf{g}$. First of all, the demand/supply balance requirement of the power grid should be met, i.e.,

$$\mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) = 0, \tag{III.10}$$

where we recall that $\mathbf{d} = \mathbf{M}\boldsymbol{\lambda}$ is the electric load due to service centers (cf. (III.4)). Second, the transmission line flow constraints under the DC approximation [20] should hold:

$$\mathbf{H}(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) \leq \mathbf{m}, \tag{III.11}$$

where the matrix $\mathbf{H}$ is explicitly defined in [20], and $\mathbf{m} = [m_f]_{f\in\mathcal{L}}$ is a vector containing transmission line flow limits.

Let us denote the cost of generating g_v units of energy at node $v\in\mathcal{V}$ as a convex and continuous function $c_v(g_v)$, and the vector of generation costs as $\boldsymbol{c}(\mathbf{g}) = [c_v(g_v)]_{v\in\mathcal{V}}$. Given a demand $\mathbf{d}$, the IPSO solves an economic dispatch problem to decide $\mathbf{g}$ [21]:

$$\begin{aligned} \min_{\mathbf{g}\succeq\mathbf{0}} \quad & \mathbf{1}^T\boldsymbol{c}(\mathbf{g}) \\ \text{s.t.} \quad & \mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) = 0, \\ & \mathbf{H}(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) \leq \mathbf{m}. \end{aligned} \tag{III.12}$$

Also, if we introduce Lagrange multipliers γ and $\boldsymbol{\mu}$ respectively for the balance and line flow constraints in (III.12), the LMP vector is given by:

$$\mathbf{p} = \gamma\mathbf{1} + \mathbf{H}^T\boldsymbol{\mu}. \tag{III.13}$$

The reader should however note that upon posting this price, the retailers in the coupled infrastructure would adjust their network flow $\boldsymbol{\lambda}^r$ according to (III.9), hence affecting the electricity demand $\mathbf{d} = \sum_{r\in\mathcal{R}}\mathbf{M}\boldsymbol{\lambda}^r$, and the optimal generation dispatch $\mathbf{g}$. This means that in order to post the price $\mathbf{p}$, modeling the response of the retailers is required. We will model and study this interaction in Section IV.

Before proceeding, we state the Lagrangian dual problem to (III.12) for later use. We adopt a quadratic generation cost function, hence writing $\mathbf{1}^T\mathbf{c}(\mathbf{g})$ as $(1/2)\mathbf{g}^T\boldsymbol{\Sigma}\mathbf{g} + \boldsymbol{b}^T\mathbf{g}$, where $\boldsymbol{\Sigma}$ is a given positive diagonal matrix. Then, the dual problem is:

$$\begin{aligned} \min_{\boldsymbol{\mu},\gamma} \quad & \tfrac{1}{2}(\mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} - \boldsymbol{b})^T\boldsymbol{\Sigma}^{-1}(\mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} - \boldsymbol{b}) \\ & \qquad - (\mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) - \mathbf{m})^T\boldsymbol{\mu} - \gamma\mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell}) \\ \text{s.t.} \quad & \boldsymbol{\mu} \geq \mathbf{0},\ \mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} \geq \boldsymbol{b}, \\ & 0 = \mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell} - \boldsymbol{\Sigma}^{-1}(\mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} - \boldsymbol{b})), \\ & \mathbf{m} - \mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) + \mathbf{H}\boldsymbol{\Sigma}^{-1}(\mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} - \boldsymbol{b}) \geq \mathbf{0}, \\ & \boldsymbol{\mu}^T(\mathbf{m} - \mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) + \mathbf{H}\boldsymbol{\Sigma}^{-1}(\mathbf{H}^T\boldsymbol{\mu} + \gamma\mathbf{1} - \boldsymbol{b})) \leq 0. \end{aligned} \tag{III.14}$$

[1]Any electric load that is exogenous to the networked infrastructure

Given the optimal dual solution $(\boldsymbol{\mu}^*, \gamma^*)$, the optimal generation pattern can be computed from the KKT condition as:

$$\mathbf{g}^* = \boldsymbol{\Sigma}^{-1}(\mathbf{H}^T\boldsymbol{\mu}^* + \gamma^*\mathbf{1} - \boldsymbol{b}). \tag{III.15}$$

IV. Socially Optimal Pricing

The retailers selfishly adjust their flow over the infrastructure network as well as as their electricity use through the power grid. Since transportation arcs and some service centers are shared resources, this leads to a non-cooperative game between these retailers. The solution concept adopted is Nash Equilibrium (NE), i.e., we seek a feasible multipolicy $\boldsymbol{\Lambda} = \{\boldsymbol{\lambda}^r\}_{r\in\mathcal{R}}$ such that all retailers costs are simultaneously minimized:

$$J^r(\boldsymbol{\Lambda}) = \min_{\tilde{\boldsymbol{\lambda}}^r \in \mathcal{F}^r} J^r(\tilde{\boldsymbol{\lambda}}^r; \boldsymbol{\lambda}^{-r}; \mathbf{p}), \forall r \in \mathcal{R} \tag{IV.1}$$

Next, we study the electricity price design problem by assuming that the service retailers are price-takers in the electricity market, i.e., they do not attemp to individually affect the market prices. This is a requirement for operation of competitive electricity markets and a common assumption in the literature.

A. Nash Equilibrium and Enforcable Flows

So how can the IPSO design the prices $\mathbf{p}$ considering the flexibility of the demand? To answer this question, we must first characterize the set of electricity demand values that can potentially result from a NE in the game between retailers.

Theorem IV.1. *The NE of the game between retailers exists [22] and is unique when $o_a \leq \frac{3|\mathcal{R}|-1}{|\mathcal{R}|-1}, \forall a \in \mathcal{E}$ [23].*

Note that o_a is the exponent of the BPR model in (III.6). The theorem simply extends results in [22], [23] on competition in network routing games to electricity price-aware route and service decisions taken in networked infrastructure systems. This extension is possible since we defined the concept of an extended infrastructure graph to capture electricity consumption decisions on a network. Note that $\forall |R| \geq 2$, $\frac{3|\mathcal{R}|-1}{|\mathcal{R}|-1} > 3$.

This NE would then lead to a certain flow on the extended graph and hence an electricity load pattern that is not necessarily socially optimal. The IPSO's goal, as described in Section III-C, is to design electricity prices to affect the electricity load of the infrastructure and lower generation costs. Now, the question is: which electricity load patterns can be enforced as a NE? This is tied to the following question: which set of flows (and hence electricity loads) can be enforced as a NE on the extended graph? To answer, we first assume that all arcs in the extended infrastructure graph accept tolls. This means that all roads in an electric transportation network or all communication links in a cloud computing network, as well entrance links into service centers, can accept tolls designed by an infrastructure system operator. In Section V, we discuss the effects of the removal of such assumptions.

Theorem IV.2. *A flow pattern $\boldsymbol{\lambda}^{\mathrm{NE}} = \{\lambda_a^{\mathrm{NE}}, \forall a \in \mathcal{E}\}$ can be enforced as a NE in the infrastructure system iff there exists an optimal solution $\boldsymbol{\lambda}^{NE}$ for the following convex optimization problem on the extended infrastructure graph such that the inequality constraint* (IV.4) *is tight:*

$$\min_{\mathbf{f}_c, c\in\mathcal{C}^r, r\in\mathcal{R}} \sum_{r\in\mathcal{R}} \Big(\sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c\Big)^T \boldsymbol{h}(\boldsymbol{\lambda}^{NE}; \mathbf{p}) + \tag{IV.2}$$

$$\frac{1}{2}\Big(\sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c\Big)^T \odot \Big(\sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c\Big)^T \boldsymbol{h}'(\boldsymbol{\lambda}^{NE}; \mathbf{p})$$

$$\text{s.t.} \quad \mathbf{f}_c \geq \mathbf{0}, \ \forall c \in \mathcal{C}^r, \tag{IV.3}$$

$$\sum_{r\in\mathcal{R}} \sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c \leq \boldsymbol{\lambda}^{\mathrm{NE}}, \tag{IV.4}$$

$$\mathbf{1}^T\mathbf{f}_c = u_c, \ \forall c \in \mathcal{C}^r, \tag{IV.5}$$

where $\boldsymbol{h}'(\boldsymbol{\lambda}; \mathbf{p}) := (h_1'(\lambda_1; \mathbf{p}), ..., h_{|\mathcal{E}|}'(\lambda_{|\mathcal{E}|}; \mathbf{p}))^T$, the derivative $h_e'(\lambda_e; \mathbf{p})$ is taken with respect to λ_e, and the operator $\odot$ denotes the Hadamard product.

Proof. The proof is similar to that of [24, Theorem 4.1]. By writing the KKT conditions of (III.9) for all r, a certain routing decision $\{\mathbf{f}_c\}_{c\in\mathcal{C}^r}$ would lead to an NE iff there exist $\boldsymbol{\xi}_c \geq \mathbf{0}$ and $\zeta_c \in \mathbb{R}$ such that:

$$\mathbf{0} = \Big(\boldsymbol{h}'(\tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r}; \mathbf{p})\mathbf{1}^T \odot \mathbf{A}_c\Big)^T \Big(\sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c\Big)$$

$$+ \mathbf{A}_c^T[\boldsymbol{h}(\tilde{\boldsymbol{\lambda}}^r + \boldsymbol{\lambda}^{-r}; \mathbf{p}) + \boldsymbol{\eta}] - \boldsymbol{\xi}_c - \zeta_c\mathbf{1}, \tag{IV.6}$$

$$\mathbf{0} = \boldsymbol{\xi}_c \odot \mathbf{f}_c,$$

for all $c \in \mathcal{C}^r, r \in \mathcal{R}$. We can verify that the conditions in (IV.6) are equivalent to the KKT conditions of the optimization (IV.2) when the inequality (IV.4) is tight. Accordingly, the optimal tolls $\boldsymbol{\eta}$ that can enforce a certain flow $\boldsymbol{\lambda}^{\mathrm{NE}}$ are given by the optimal Lagrange multipliers associated with the constraint (IV.4). □

The next question is to characterize the optimal electricity load that we would ideally like to enforce in the infrastructure, along with the electricity price that can enforce it.

B. Socially Optimal Electricity Prices

The overall costs of the infrastructure system and the power grid can be jointly optimized by solving the following problem:

$$\min_{\mathbf{F}, \mathbf{g}} \boldsymbol{\lambda}^T \boldsymbol{s}(\boldsymbol{\lambda}) + \mathbf{1}^T \boldsymbol{c}(\mathbf{g}) \tag{IV.7}$$

$$\text{s.t.} \quad \star \begin{cases} \mathbf{f}_c \geq \mathbf{0}, \ \forall c \in \mathcal{C}^r, \\ \mathbf{1}^T\mathbf{f}_c = u_c, \ \forall c \in \mathcal{C}^r, \\ \boldsymbol{\lambda} = \sum_{r\in\mathcal{R}} \boldsymbol{\lambda}^r \\ \boldsymbol{\lambda}^r = \sum_{c\in\mathcal{C}^r} \mathbf{A}_c\mathbf{f}_c, \end{cases}$$

$$\dagger \begin{cases} \mathbf{g}^{\min} \leq \mathbf{g} \leq \mathbf{g}^{\max}, \\ \mathbf{1}^T(\mathbf{M}\boldsymbol{\lambda} + \boldsymbol{\ell} - \mathbf{g}) = 0, \\ \mathbf{H}(\mathbf{M}\boldsymbol{\lambda} + \boldsymbol{\ell} - \mathbf{g}) \leq \mathbf{m}, \end{cases}$$

This optimal policy would lead to an optimal flow dispatch $\boldsymbol{\lambda}^*$ and generation $\mathbf{g}^*$. Now remember that each retailer r would incur a cost of:

$$J^r(\boldsymbol{\Lambda}^*) = (\boldsymbol{\lambda}^{r,*})^T \left[\boldsymbol{s}\left(\boldsymbol{\lambda}^{r,*} + \boldsymbol{\lambda}^{-r,*}\right) + \mathbf{M}^T \mathbf{p}^* + \boldsymbol{\eta}^* \right]. \tag{IV.8}$$

So, the question is: does there exist an electricity price $\mathbf{p}^*$ and a toll vector $\boldsymbol{\eta}^*$ that can enforce $\boldsymbol{\lambda}^*$ in the infrastructure and power systems? We answer this question in two steps.

Proposition IV.3. *Let us first assume that the optimal flow $\boldsymbol{\lambda}^*$ can be enforced in the infrastructure system. Then, the efficient market clearing LMP $\mathbf{p}^*$ can be calculated through a collaboration between the IPSO and the infrastructure system operator without the operators sharing their system data with each other. This is facilitated through a dual decomposition based algorithm.*

Proof sketch: It is straightforward to show that (IV.7) can be split into two subproblems, with the first being:

$$\begin{aligned} \min_{f_c^k, c\in\mathcal{C}^r, k\in\mathcal{K}_c} \quad & \boldsymbol{\lambda}^T \boldsymbol{s}(\boldsymbol{\lambda}) + \mathbf{p}^T \mathbf{M}\boldsymbol{\lambda} \\ \text{s.t.} \quad & \text{Constraints marked by } (\star) \text{ in (IV.7)} \end{aligned} \tag{IV.9}$$

and the second being the generation dispatch optimization shown in (III.12), with $\mathbf{p}$ calculated as in (III.13). By iteratively updating the value of $\mathbf{p}$ according to the dual decomposition framework, the solution of these two subproblems will converge to that of (IV.7). This can allow the IPSO and the infrastructure system operators to not share their system data with each other in order to calculate the optimal prices of electricity [16].

Hence, if we assume that one can enforce the solution of (IV.9) as a NE between the retailers, and given that the IPSO's problem is enforceable through LMPs, the optimal flow and generation dispatch can be enforced.

So the problem that remains is whether the solution of optimization problem (IV.9) is enforceable as a NE, which we address in the following proposition.

Proposition IV.4. *The optimal solution of* (IV.9) *can be enforced in the infrastructure system, and the optimal tolls $\boldsymbol{\eta}^*$ can be calculated through* (IV.2)*.*

Proof. Denote the optimal solution of (IV.9) as $\mathbf{F}^{\text{opt}}$. The basic idea behind the proof is similar to that of [25] and is as follows: For any feasible flow $\mathbf{F}^{\text{opt}}$, one can obtain a flow $\mathbf{Q} \leq \mathbf{F}^{\text{opt}}$ such that every feasible solution, and hence any optimal solution, to (IV.2) must satisfy all constraints in (IV.2) with equality. According to Theorem IV.2, such a $\mathbf{Q}$ is enforceable. If $\mathbf{F}^{\text{opt}}$ is an optimal solution of (IV.9), then so is $\mathbf{Q}$; thus, the optimal solution of (IV.9) is enforcable. The optimal tolls to enforce this flow are the optimal Lagrange multipliers $\boldsymbol{\eta}^*$ in (IV.2) for the flow $\mathbf{Q}$. □

V. MPEC Formulation Considering Potential Toll Constraints

When the service centers are privately owned by retailers, there may not be any straightforward financial mechanism to toll their virtual entrance arcs in most infrastructures. This problem potentially extends to transportation arcs, particularly in data networks, where many communication links are privately owned. This means that the optimal solution of (IV.9) can no longer be enforced. Hence, we must be able to find the best electricity demand that can be enforced without imposing tolls on transportation and virtual entrance arcs.

With the congestion in the infrastructure system not being a concern to the IPSO, the electricity price design problem can be thought of as a bilevel optimization problem. At the upper level, the IPSO designs the electricity price and minimizes the cost of generation (cf. (III.12)); while the lower level problem characterizes the possible set of electricity loads that can be the result of a NE in the infrastructure.

In particular, using the notation defined in (III.13), the bilevel optimization problem can be cast as:

$$\begin{aligned} \min_{\mathbf{g},\mathbf{p}} \quad & \mathbf{1}^T \mathbf{c}(\mathbf{g}) \\ \text{s.t.} \quad & \mathbf{g} \geq \mathbf{0},\ \mathbf{p} = \mathbf{H}^T \boldsymbol{\mu} + \gamma \mathbf{1}, \\ & \gamma : \mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) = \mathbf{0},\ \boldsymbol{\mu} : \mathbf{H}(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) \leq \boldsymbol{m}, \\ & \mathbf{d} = \boldsymbol{M} \textstyle\sum_{r\in\mathcal{R}} \boldsymbol{\lambda}^r, \\ & \forall\, r \in \mathcal{R} : \boldsymbol{\lambda}^r = \arg \min_{\tilde{\boldsymbol{\lambda}}^r \in \mathcal{F}^r} J(\tilde{\boldsymbol{\lambda}}^r; \boldsymbol{\lambda}^{-r}; \mathbf{p}). \end{aligned} \tag{V.1}$$

We remark that this can be compared to the *greedy pricing* approach studied in [16]. In particular, the greedy pricing approach attempts to solve (V.1) by alternatively solving the upper and lower level problems. It has been shown that this approach may results in a limit-cycle type of oscillation in electricity prices, hence never converging to a stable price.

Through transforming the upper level problem into its dual form (see (III.14)) and studying the KKT conditions of the lower-level problem (see (IV.6)), problem (V.1) can be expressed as follows:

$$\begin{aligned} \min \quad & \frac{1}{2}(\mathbf{p} - \boldsymbol{b})^T \boldsymbol{\Sigma}^{-1} (\mathbf{p} - \boldsymbol{b}) \qquad\qquad (\text{V.2}) \\ & - (\mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) - \mathbf{m})^T \boldsymbol{\mu} - \gamma \mathbf{1}^T (\mathbf{d} + \boldsymbol{\ell}) \\ \text{w.r.t.} \quad & \mathbf{p}, \boldsymbol{x}, \boldsymbol{\mu}, \gamma, \mathbf{d}, \boldsymbol{\lambda}, \{\mathbf{f}_c, \boldsymbol{\xi}_c, \zeta_c\}_{c\in\mathcal{C}^r, r\in\mathcal{R}} \\ \text{s.t.} \quad & \boldsymbol{\mu} \geq \mathbf{0},\ \boldsymbol{x} \geq \mathbf{0},\ \mathbf{H}^T \boldsymbol{\mu} + \gamma \mathbf{1} \geq \boldsymbol{b},\ \mathbf{p} = \mathbf{H}^T \boldsymbol{\mu} + \gamma \mathbf{1}, \\ & 0 = \mathbf{1}^T (\mathbf{d} + \boldsymbol{\ell} - \boldsymbol{\Sigma}^{-1}(\mathbf{p} - \boldsymbol{b})), \\ & \boldsymbol{x} = \mathbf{m} - \mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) + \mathbf{H}\boldsymbol{\Sigma}^{-1}(\mathbf{p} - \boldsymbol{b}), \\ & \boldsymbol{x} \odot \boldsymbol{\mu} = 0, \qquad\qquad (\text{V.3}) \\ & \boldsymbol{\lambda} = \textstyle\sum_{r\in\mathcal{R}} \sum_{c\in\mathcal{C}^r} \mathbf{A}_c \mathbf{f}_c,\ \mathbf{d} = \mathbf{M}\boldsymbol{\lambda}, \\ & \mathbf{0} = \mathbf{A}_c^T (\boldsymbol{h}(\boldsymbol{\lambda}; \mathbf{p}) + \boldsymbol{\eta}) + (\boldsymbol{h}'(\boldsymbol{\lambda}; \mathbf{p}) \mathbf{1}^T \odot \mathbf{A}_c)^T \boldsymbol{\lambda}, \\ & \quad - \boldsymbol{\xi}_c - \zeta_c \mathbf{1},\ \forall\, c \in \mathcal{C}^r,\ \forall\, r \in \mathcal{R}, \qquad\qquad (\text{V.4}) \\ & \mathbf{0} = \boldsymbol{\xi}_c \odot \mathbf{f}_c,\ \mathbf{f}_c \geq \mathbf{0},\ \boldsymbol{\xi}_c \geq \mathbf{0},\ \mathbf{1}^T \mathbf{f}_c = u_c, \qquad\qquad (\text{V.5}) \\ & \quad \forall\, c \in \mathcal{C}^r,\ \forall\, r \in \mathcal{R}. \end{aligned}$$

Solving the above problem is non-trivial due to the non-convexity in the objective function and the constraints (V.3) – (V.5). While the upper and lower level problems are convex individually, the non-convexity arises as the optimization variables of the upper and lower level problems are coupled together. These are commonly known as the *equilibrium* *constraints* and (V.2) is thus a mathematical program with equilibrium constraints (MPEC) [26].

In our case of interest, for the BPR delay function in (III.6), we consider the parameter $o_a = 1$ for all a. As such, the element wise derivatives $h'_a(\lambda_a; \mathbf{p})$ are equal to 1 for all λ_a. The equality (V.4) is thus convex and can be written as:

$$\mathbf{0} = \mathbf{A}_c^T(\boldsymbol{h}(\boldsymbol{\lambda}; \mathbf{p}) + \boldsymbol{\lambda}) - \boldsymbol{\xi}_c - \zeta_c \mathbf{1}. \tag{V.6}$$

To handle the non-convex complementary constraint in (V.2), we resort to a mixed-integer-programming based approach [27]. Observe that (V.2) is equivalent to the following:

$$\begin{aligned}
\min\ & \frac{1}{2}(\mathbf{p} - \boldsymbol{b})^T \boldsymbol{\Sigma}^{-1}(\mathbf{p} - \boldsymbol{b}) \qquad \text{(V.7)}\\
& - (\mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) - \mathbf{m})^T \boldsymbol{\mu} - \gamma \mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell})\\
\text{w.r.t. } & \mathbf{p}, \boldsymbol{\mu}, \boldsymbol{x}, \mathbf{z}_x, \gamma, \mathbf{d}, \boldsymbol{\lambda}, \{\mathbf{f}_c, \boldsymbol{\xi}_c, \zeta_c, \mathbf{z}_c\}_{c \in \mathcal{C}^r, r \in \mathcal{R}}\\
\text{s.t. } & \boldsymbol{\mu} \geq \mathbf{0},\ \boldsymbol{x} \geq \mathbf{0},\ \mathbf{H}^T \boldsymbol{\mu} + \gamma \mathbf{1} \geq \boldsymbol{b},\ \mathbf{p} = \mathbf{H}^T \boldsymbol{\mu} + \gamma \mathbf{1},\\
& 0 = \mathbf{1}^T(\mathbf{d} + \boldsymbol{\ell} - \boldsymbol{\Sigma}^{-1}(\mathbf{p} - \boldsymbol{b})),\\
& \boldsymbol{x} = \mathbf{m} - \mathbf{H}(\mathbf{d} + \boldsymbol{\ell}) + \mathbf{H}\boldsymbol{\Sigma}^{-1}(\mathbf{p} - \boldsymbol{b}),\\
& \boldsymbol{x} \leq L\mathbf{z}_x,\ \boldsymbol{\mu} \leq L(1 - \mathbf{z}_x),\ \mathbf{z}_x \in \{0, 1\}^{|\mathcal{F}|},\\
& \boldsymbol{\lambda} = \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}^r} \mathbf{A}_c \mathbf{f}_c,\ \mathbf{d} = \mathbf{M}\boldsymbol{\lambda},\\
& \mathbf{0} = \mathbf{A}_c^T(\boldsymbol{h}(\boldsymbol{\lambda}; \mathbf{p}) + \boldsymbol{\eta} + \boldsymbol{\lambda}) - \boldsymbol{\xi}_c - \zeta_c \mathbf{1},\\
& \mathbf{f}_c \geq \mathbf{0},\ \boldsymbol{\xi}_c \geq \mathbf{0},\ \mathbf{1}^T \mathbf{f}_c = u_c,\\
& \mathbf{f}_c \leq L \cdot \mathbf{z}_c,\ \boldsymbol{\xi}_c \leq L \cdot (\mathbf{1} - \mathbf{z}_c),\\
& \mathbf{z}_c \in \{0, 1\}^{|\mathcal{K}_c|},\ \forall\ c \in \mathcal{C}^r,\ \forall\ r \in \mathcal{R},
\end{aligned}$$

where $L > 0$ is some sufficiently large number.

However, (V.7) is not a mixed integer program due to non-convexity in the objective function. As a remedy, we apply the successive convex approximation technique. We use the fact that given $(\mathbf{d}^k, \boldsymbol{\mu}^k, \gamma^k)$, the following upper bound holds:

$$\begin{aligned}
-(\mathbf{H}\mathbf{d})^T \boldsymbol{\mu} \leq \frac{1}{4}\Big(& \|\mathbf{H}\mathbf{d} - \boldsymbol{\mu}\|_2^2 - \|\mathbf{H}\mathbf{d}^k + \boldsymbol{\mu}^k\|_2^2\\
& - 2((\boldsymbol{\mu} - \boldsymbol{\mu}^k) + \mathbf{H}(\mathbf{d} - \mathbf{d}^k))^T(\mathbf{H}\mathbf{d}^k + \boldsymbol{\mu}^k)\Big).
\end{aligned} \tag{V.8}$$

Similarly, we can derive a convex upper bound for the other bilinear terms in the objective function. This yields a mixed integer program with convex objective and constraints. The MPEC problem (V.7) can be tackled using a successive convex approximation technique, as shown in Algorithm 1.

Note that the infrastructure system's costs at this NE flow $\boldsymbol{\lambda}^{NE}$ may be far from the system optimal solution $\boldsymbol{\lambda}^*$ in (IV.7). To study this effect, we update bounds on the so-called *Price of Anarchy* (PoA) for atomic network routing games in [28]. This bound on the PoA determines how much the infrastructure system users can lose in terms of efficiency due to selfish behavior (with lack of tolls), considering the new coupling with the power grid. In other words, it provides a bound on how much the objective function value in (IV.9) can increase.

Proposition V.1. *The price of anarchy ρ for the infrastructure with no tolls imposed on the arcs of the extended infrastructure graph is bounded by:*

$$1 \leq \rho = \frac{\left(\boldsymbol{\lambda}^{NE}\right)^T \Big[\boldsymbol{s}\left(\boldsymbol{\lambda}^{NE}\right) + \mathbf{M}^T \mathbf{p}^{NE}\Big]}{(\boldsymbol{\lambda}^*)^T \left[\boldsymbol{s}(\boldsymbol{\lambda}^*) + \mathbf{M}^T \mathbf{p}^*\right]} \leq \frac{1}{1 - \xi}, \tag{V.9}$$

where $\xi = \max_{a \in \mathcal{E}} \xi_a$ and

$$\begin{aligned}
\xi_a =\ & o_a \left[\left(\frac{(o_a \zeta_a + 1)^{\frac{1}{o_a}}}{(1 + o_a)^{\frac{1}{o_a}} - \chi_a}\right) - \zeta_a\right] \zeta_a - o_a \frac{(1 - \zeta_a)^2}{|\mathcal{R}| - 1}\\
& + (1 - \zeta_a) \frac{o_a}{1 + o_a} \left(\frac{(o_a \zeta_a + 1)^{\frac{1}{o_a}}}{(1 + o_a)^{\frac{1}{o_a}} - \chi_a}\right),
\end{aligned} \tag{V.10}$$

$$\zeta_a = \frac{\max_{r \in \mathcal{R}} \lambda_a^{r,NE}}{\lambda_a^{NE}},\ \chi_a = \sum_{b \in \mathcal{B}} [\mathbf{M}]_{b,a}(p_b^{NE} - p_b^{min}), \tag{V.11}$$

where p_b^{min} is the minimum electricity price at bus b (taking the infrastructure load as zero).

Proof. The new coupling with the power grid affects equation (9) in the proof outlined in [28], leading the the new term χ_a. This new term provides a bound on increase in electricity costs. □

Algorithm 1 Successive Convex Approximation (SCA) for solving the MPEC problem (V.2) for price design.

Initialize: $(\mathbf{d}^0, \boldsymbol{\mu}^0, \gamma^0)$;
for $k = 1, 2, \ldots$ **do**
  Given $(\mathbf{d}^{k-1}, \boldsymbol{\mu}^{k-1}, \gamma^{k-1})$, solve the integer program (V.7) by replacing its objective function with the convex upper bound surrogate (cf. (V.8)). Take the respective optimal solution as $(\mathbf{d}^k, \boldsymbol{\mu}^k, \gamma^k)$.
end for
Return: the optimal pricing $\mathbf{p}^k$ and the generation pattern $\mathbf{g}^k$ (computed from (III.15)).

VI. Tackling the Dimensionality Problem in Large-Scale Networks

Here, we describe a preprocessing technique to help reduce the complexity of solving (III.9) in large-scale networks. Define a covering of $\mathcal{K}_c$ as $\cup_{t \in \mathcal{K}_c} \mathcal{K}_c^t$. The paths in the same subset $\mathcal{K}_c^t$ share the same transportation arcs and entrance arcs (i.e., stop for service at the same service centers), but can choose different service options at each service center. The set $\mathcal{K}_c$ includes as its members any sets of arcs on the extended graph that can be extended into a path in $\mathcal{K}_c$ solely through the addition of virtual service arcs to the set.

Proposition VI.1. *One dominating path in each subset $\mathcal{K}_c^t$ is sufficient to represent all paths in $\mathcal{K}_c^t$ in both socially optimal solutions as well as any selfish user equilibrium.*

Proof. This follows from the definition that the cost of traveling each virtual arc a located at a service center at node v is a constant $(p_v e_a + \beta_a)$ and does not change with the flow on the arc. Hence, selfish user selections would coincide with socially optimal selections in the subnetwork of virtual

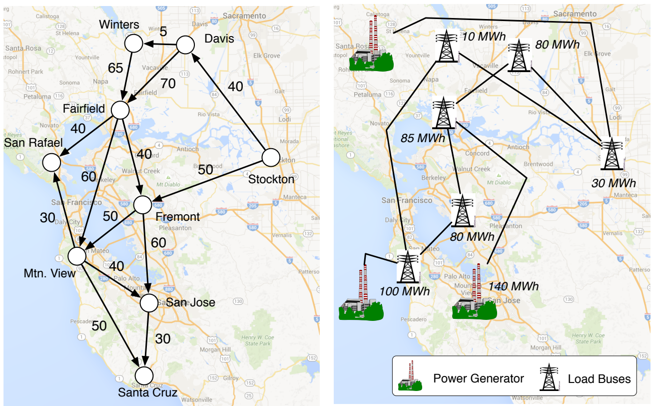

Fig. 3. (Left) Transportation network with the traveling time (in minutes) on arcs. (Right) Power network labelled with the base load at each bus. Notice that the nodes ‘San Jose’, ‘Santa Cruz’ and ‘San Rafael’ are not modeled with fast charging capabilities as these are the set of destination nodes.

service arcs. Thus, electricity costs can be preoptimized and act as constant mark-ups for each possible combination of service centers that a job is sent to, and all paths $k \in \mathcal{K}_c^t$ can be represented through a single dominant path. $\square$

Hence, we can simply reduce the size of the set $\mathcal{K}_c$ through an offline calculation for each given price $\mathbf{p}$.

VII. Numerical Results

We consider an application of the proposed joint optimization strategies to a fictitious network, modeled after the San Francisco bay area, as shown in Figure 3; while the power network is modeled after the IEEE-9 bus test case. We assume that the retailers are owners of charging networks responsible for routing electric vehicles (EVs) traveling between different origin-destination pairs. There are $|\mathcal{R}| = 3$ retailers in the network, each advising a group of EVs. For each retailer, there is one class of EVs with $u_c = 500$ and the drivers of the EVs desire to travel (i) from Stockton to San Rafael, (ii) from Davis to Santa Cruz and (iii) from Davis to San Jose. All the EVs are equipped with 20 kWh of initial charge and each fast charging station allows the EVs to charge for 0 kWh, 10 kWh or 20 kWh. The battery capacity of an EV is 60 kWh. Each EV consumes 10 kWh of charge to travel for 30 minutes.

We compare the total infrastructure cost (electricity and congestion) for the socially optimal (SO) strategy (IV.7) and that of the Successive Convex Approximation (SCA) technique in Algorithm 1. In particular, we solve the convex approximate mixed-integer second order cone program (V.7) using the solver `gurobi` in MATLAB. The simulation results are presented in Figure 4. As seen, the objective value of the SCA algorithm converges rapidly. The converged solution is clearly suboptimal to the optimal flow problem (IV.7) as the converged solution from Algorithm 1 corresponds to best possible cost with the retailers’ decisions at a NE with no tolls. We also evaluate the bound on price of anarchy (PoA) using Eq. (V.9). In the considered case, the bound on the PoA

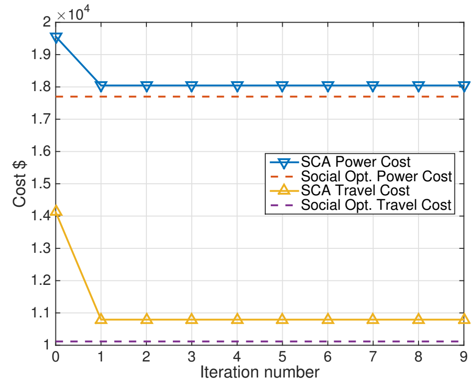

Fig. 4. Total cost of the SCA strategy against the iteration number. Notice that Theorem V.1 bounds the ratio between the travel cost found by solving for the NE using (V.7) and travel cost at the social optimum.

is found to be 1.4665. This indicates that the NE solution yields an infrastructure cost that is at most 1.4665 times the social optimum cost. This is corroborated in Figure 4 as well.

VIII. Conclusions and Future Work

We studied the problem of optimal electricity prices in the presence of large networked infrastructure systems with flexible electricity demand. We believe that our work is a natural first step toward a theoretical treatment of this complex problem. The model adopted was rather stylized and not fine-tuned for any of the specific infrastructures that were presented as a motivating example. Also, temporal dynamics common to transportation networks can be captured via a more notation-heavy formulation that we leave to future work. We note that dissipative flows common to gas and water networks cannot be captured under the model proposed here and we refer the reader to [29] for a detailed treatment of this type of network flows. Furthermore, finding the solution of the realistic economic dispatch problem (V.1) requires complex bid formats from the retailers that we plan to study further in the future.

References

[1] D. Papadaskalopoulos and G. Strbac, “Decentralized participation of flexible demand in electricity markets; part i: Market mechanism,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3658–3666, Nov 2013.

[2] L. Wu, “Impact of price-based demand response on market clearing and locational marginal prices,” *IET Gener. Transm. Distrib.*, vol. 7, pp. 1087–1095(8), Oct 2013.

[3] L. Chen, N. Li, L. Jiang, and S. H. Low, “Optimal demand response: problem formulation and deterministic case,” in *Control and optimization methods for electric smart grids*. Springer, 2012, pp. 63–85.

[4] M. Parvania, M. Fotuhi-Firuzabad, and M. Shahidehpour, “Optimal demand response aggregation in wholesale electricity markets,” *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1957–1965, Dec 2013.

[5] C. De Jonghe, B. Hobbs, and R. Belmans, “Optimal generation mix with short-term demand response and wind penetration,” *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 830–839, May 2012.

[6] D. T. Nguyen, M. Negnevitsky, and M. de Groot, “Pool-based demand response exchange;concept and modeling,” *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1677–1685, 2011.

[7] Z. Liu, A. Wierman, Y. Chen, B. Razon, and N. Chen, “Data center demand response: Avoiding the coincident peak via workload shifting and local generation,” *Performance Evaluation*, vol. 70, no. 10, pp. 770–791, 2013.

[8] X. Yin and B. Sinopoli, "Adaptive robust optimization for coordinated capacity and load control in data centers," in *53rd Annual Conference on Decision and Control (CDC)*. IEEE, 2014, pp. 5674–5679.
[9] Z. Liu, M. Lin, A. Wierman, S. H. Low, and L. L. Andrew, "Greening geographical load balancing," in *Proc. of ACM SIGMETRICS '11*, 2011, pp. 233–244.
[10] L. Rao, X. Liu, L. Xie, and W. Liu, "Minimizing electricity cost: Optimization of distributed internet data centers in a multi-electricity-market environment," in *Proc. of IEEE INFOCOM*, 2010.
[11] S. Shao, M. Pipattanasomporn, and S. Rahman, "Demand response as a load shaping tool in an intelligent grid with electric vehicles," *IEEE Trans. Smart Grid*, vol. 2, no. 4, pp. 624–631, Dec 2011.
[12] I. Bayram, G. Michailidis, M. Devetsikiotis, and F. Granelli, "Electric power allocation in a network of fast charging stations," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 7, pp. 1235–1246, July 2013.
[13] I. S. Bayram, G. Michailidis, M. Devetsikiotis, F. Granelli, and S. Bhattacharya, "Smart vehicles in the smart grid: Challenges, trends, and application to the design of charging stations," in *Control and Opt. Methods for Elec. Smart Grids*. Springer, 2012, pp. 133–145.
[14] M. Khodayar, L. Wu, and Z. Li, "Electric vehicle mobility in transmission-constrained hourly power generation scheduling," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 779–788, June 2013.
[15] M. Alizadeh, H.-T. Wai, A. Scaglione, A. Goldsmith, Y. Fan, and T. Javidi, "Optimized path planning for electric vehicle routing and charging," in *Allerton Conference*, 2014.
[16] M. Alizadeh, H.-T. Wai, M. Chowdhury, A. Goldsmith, A. Scaglione, and T. Javidi, "Optimal pricing to manage electric vehicles in coupled power and transportation networks," *IEEE Transactions on Control of Network Systems*, 2016.
[17] F. He, Y. Yin, and J. Zhou, "Integrated pricing of roads and electricity enabled by wireless power transfer," *Transportation Research Part C: Emerging Technologies*, vol. 34, pp. 1 – 15, 2013.
[18] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot pricing of electricity*. Springer, 2013.
[19] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, "Network flows," DTIC Document, Tech. Rep., 1988.
[20] A. Verma, "Power grid security analysis: An optimization approach," Ph.D. dissertation, Columbia University, 2009.
[21] J. D. Glover, M. Sarma, and T. Overbye, *Power System Analysis & Design*. Cengage Learning, 2011.
[22] A. Orda, R. Rom, and N. Shimkin, "Competitive routing in multiuser communication networks," *IEEE/ACM Trans. on Networking (ToN)*, vol. 1, no. 5, pp. 510–521, 1993.
[23] E. Altman, T. Başar, T. Jimenez, and N. Shimkin, "Competitive routing in networks with polynomial costs," *IEEE Trans. Autom. Control*, vol. 47, no. 1, pp. 92–96, 2002.
[24] C. Swamy, "The effectiveness of stackelberg strategies and tolls for network congestion games," in *Proc. of ACM-SIAM SODA*, 2007, pp. 1133–1142.
[25] L. Fleischer, K. Jain, and M. Mahdian, "Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games," in *Proc. of 45th FOCS*. IEEE, 2004, pp. 277–285.
[26] B. F. Hobbs, C. B. Metzler, and J.-S. Pang, "Strategic gaming analysis for electric power systems: An mpec approach," *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 638–645, 2000.
[27] D. S. Wicaksono, I. A. Karimi, "Piecewise MILP under- and overestimators for global optimization of bilinear programs," *AIChE Journal*, vol. 54, no. 4, pp. 991–1008, 2008.
[28] H. Yang, D. Han, and H. K. Lo, "Efficiency of atomic splittable selfish routing with polynomial cost functions," *Networks and Spatial Economics*, vol. 8, no. 4, pp. 443–451, 2008.
[29] M. Chertkov, M. Fisher, S. Backhaus, R. Bent, and S. Misra, "Pressure fluctuations in natural gas networks caused by gas-electric coupling," in *48th HICSS*. IEEE, 2015, pp. 2738–2747.