

Units and Dimensions



In 1960 the International System of Units was given official status at the Eleventh General Conference on weights and measures held in Paris, France. This system of units is an expanded version of the rationalized meter-kilogram-second-ampere (MKSA) system of units and is based on six fundamental or basic units. The six basic units are the units of length, mass, time, current, temperature, and luminous intensity.

The international unit of length is the meter. It is exactly 1,650,763.73 times the wavelength in vacuum of the radiation corresponding to the unperturbed transition between the levels $2p_{10}$ and $5d_5$ of the atom of krypton-86, the orange-red line. The international unit of mass is the kilogram. It is the mass of the International Prototype Kilogram which is a particular cylinder of platinum-iridium alloy preserved in a vault at Sevres, France, by the International Bureau of Weights and Measures. The international unit of time is the second. It is equal to 9,192,631,770 times the period corresponding to the frequency of the transition between the hyperfine levels $F = 4, M = 0$ and $F = 3, M = 0$ of the fundamental state $^2S_{1/2}$ of the cesium-133 atom unperturbed by external fields.

To present the definition for the international unit of current, we first define the newton, which is the unit of force, derived from the fundamental units meter, kilogram, and second in the following manner. Since velocity is rate of change of distance with time, its unit is meter per second. Since acceleration is rate of change of velocity with time, its unit is meter per second per second or meter per second squared. Since force is mass times acceleration, its unit is kilogram-meter per second squared, also known as the newton. Thus, the newton is that force which imparts an acceleration of 1 meter per second squared to a mass of 1 kilogram. The international unit of current, which is the ampere, can now be defined. It is the constant current that when maintained in two straight, infinitely long, parallel conductors of negligible cross section and placed 1 meter apart in vacuum produces a force of 2×10^{-7} newtons per meter length of the conductors.

The international unit of temperature is the Kelvin degree. It is based on the definition of the thermodynamic scale of temperature by designating the triple-point of water as a fixed fundamental point to which a temperature of exactly 273.16 degrees Kelvin is attributed. The international unit of luminous intensity is the candela. It is

defined such that the luminance of a blackbody radiator at the freezing temperature of platinum is 60 candelas per square centimeter.

We have just defined the six basic units of the International System of Units. Two supplementary units are the radian and the steradian for plane angle and solid angle, respectively. All other units are derived units. For example, the unit of charge, the coulomb, is the amount of charge transported in 1 second by a current of 1 ampere; the unit of energy, the joule, is the work done when the point of application of a force of 1 newton is displaced a distance of 1 meter in the direction of the force; the unit of power, the watt, is the power that gives rise to the production of energy at the rate of 1 joule per second; the unit of electric potential difference, the volt, is the difference of electric potential between two points of a conducting wire carrying constant current of 1 ampere when the power dissipated between these points is equal to 1 watt; and so on. The units for the various quantities used in this book are listed in Table C.1, together with the symbols of the quantities and their dimensions.

TABLE C.1 Symbols, Units, and Dimensions of Various Quantities

| Quantity | Symbol | Unit (symbol) | Dimensions |
|------------------------------|---|--|----------------------|
| Admittance | \bar{Y} | siemens (S) | $M^{-1}L^{-2}TQ^2$ |
| Area | A | square meter (m ²) | L^2 |
| Attenuation constant | α | neper/meter (Np/m) | L^{-1} |
| Capacitance | C | farad (F) | $M^{-1}L^{-2}T^2Q^2$ |
| Capacitance per unit length | \mathcal{C} | farad/meter (F/m) | $M^{-1}L^{-3}T^2Q^2$ |
| Cartesian coordinates | $\begin{cases} x \\ y \\ z \end{cases}$ | meter (m) | L |
| | | meter (m) | L |
| | | meter (m) | L |
| Characteristic admittance | Y_0 | siemens (S) | $M^{-1}L^{-2}TQ^2$ |
| Characteristic impedance | Z_0 | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Charge | Q, q | coulomb (C) | Q |
| Conductance | G | siemens (S) | $M^{-1}L^{-2}TQ^2$ |
| Conductance per unit length | \mathcal{G} | siemens/meter (S/m) | $M^{-1}L^{-3}TQ^2$ |
| Conduction current density | \mathbf{J}_c | ampere/square meter (A/m ²) | $L^{-2}T^{-1}Q$ |
| Conductivity | σ | siemens/meter (S/m) | $M^{-1}L^{-3}TQ^2$ |
| Current | I | ampere (A) | $T^{-1}Q$ |
| Cutoff frequency | f_c | hertz (Hz) | T^{-1} |
| Cutoff wavelength | λ_c | meter (m) | L |
| Cylindrical coordinates | $\begin{cases} r, r_c \\ \phi \\ z \end{cases}$ | meter (m) | L |
| | | radian (rad) | — |
| | | meter (m) | L |
| Differential length element | $d\mathbf{l}$ | meter (m) | L |
| Differential surface element | $d\mathbf{S}$ | square meter (m ²) | L^2 |
| Differential volume element | dv | cubic meter (m ³) | L^3 |
| Directivity | D | — | — |
| Displacement flux density | \mathbf{D} | coulomb/square meter (C/m ²) | $L^{-2}Q$ |
| Electric dipole moment | \mathbf{p} | coulomb-meter (C-m) | LQ |
| Electric field intensity | \mathbf{E} | volt/meter (V/m) | $MLT^{-2}Q^{-1}$ |
| Electric potential | V | volt (V) | $ML^2T^{-2}Q^{-1}$ |
| Electric susceptibility | χ_e | — | — |
| Electron density | N | (meter) ⁻³ (m ⁻³) | L^{-3} |
| Electronic charge | e | coulomb (C) | Q |
| Energy | W | joule (J) | ML^2T^{-2} |

TABLE C.1 Continued

| Quantity | Symbol | Unit (symbol) | Dimensions |
|--------------------------------|---|---|----------------------|
| Energy density | w | joule/cubic meter (J/m ³) | $ML^{-1}T^{-2}$ |
| Force | \mathbf{F} | newton (N) | MLT^{-2} |
| Frequency | f | hertz (Hz) | T^{-1} |
| Group velocity | v_g | meter/second (m/s) | LT^{-1} |
| Guide characteristic impedance | η_g | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Guide wavelength | λ_g | meter (m) | L |
| Impedance | \bar{Z} | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Inductance | L | henry (H) | ML^2Q^{-2} |
| Inductance per unit length | \mathcal{L} | henry/meter (H/m) | MLQ^{-2} |
| Intrinsic impedance | η | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Length | l | meter (m) | L |
| Line charge density | ρ_L | coulomb/meter (C/m) | $L^{-1}Q$ |
| Magnetic dipole moment | \mathbf{m} | ampere-square meter (A·m ²) | $L^2T^{-1}Q$ |
| Magnetic field intensity | \mathbf{H} | ampere/meter (A/m) | $L^{-1}T^{-1}Q$ |
| Magnetic flux | ψ | weber (Wb) | $ML^2T^{-1}Q^{-1}$ |
| Magnetic flux density | \mathbf{B} | tesla or weber/square meter (T or Wb/m ²) | $MT^{-1}Q^{-1}$ |
| Magnetic susceptibility | χ_m | — | — |
| Magnetic vector potential | \mathbf{A} | weber/meter (Wb/m) | $MLT^{-1}Q^{-1}$ |
| Magnetization current density | \mathbf{J}_m | ampere/square meter (A/m ²) | $L^{-2}T^{-1}Q$ |
| Magnetization vector | \mathbf{M} | ampere/meter (A/m) | $L^{-1}T^{-1}Q$ |
| Mass | m | kilogram (kg) | M |
| Mobility | μ | square meter/volt-second (m ² /V·s) | $M^{-1}TQ$ |
| Permeability | μ | henry/meter (H/m) | MLQ^{-2} |
| Permeability of free space | μ_0 | henry/meter (H/m) | MLQ^{-2} |
| Permittivity | ϵ | farad/meter (F/m) | $M^{-1}L^{-3}T^2Q^2$ |
| Permittivity of free space | ϵ_0 | farad/meter (F/m) | $M^{-1}L^{-3}T^2Q^2$ |
| Phase constant | β | radian/meter (rad/m) | L^{-1} |
| Phase velocity | v_p | meter/second (m/s) | LT^{-1} |
| Plasma frequency | f_N | hertz (Hz) | T^{-1} |
| Polarization current density | \mathbf{J}_p | ampere/square meter (A/m ²) | $L^{-2}T^{-1}Q$ |
| Polarization vector | \mathbf{P} | coulomb/square meter (C/m ²) | $L^{-2}Q$ |
| Power | P | watt (W) | ML^2T^{-3} |
| Power density | p | watt/square meter (W/m ²) | MT^{-3} |
| Poynting vector | \mathbf{P} | watt/square meter (W/m ²) | MT^{-3} |
| Propagation constant | $\bar{\gamma}$ | (meter) ⁻¹ (m ⁻¹) | L^{-1} |
| Propagation vector | $\hat{\beta}$ | radian/meter (rad/m) | L^{-1} |
| Radian frequency | ω | radian/second (rad/s) | T^{-1} |
| Radiation resistance | R_{rad} | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Reactance | X | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Reflection coefficient | Γ | — | — |
| Refractive index | n | — | — |
| Relative permeability | μ_r | — | — |
| Relative permittivity | ϵ_r | — | — |
| Resistance | R | ohm (Ω) | $ML^2T^{-1}Q^{-2}$ |
| Shielding factor | S | — | — |
| Skin depth | δ | meter (m) | L |
| Spherical coordinates | $\left\{ \begin{array}{l} r, r_s \\ \theta \\ \phi \end{array} \right.$ | meter (m) | L |
| | | radian (rad) | — |
| | | radian (rad) | — |

(continued)

TABLE C.1 Continued

| Quantity | Symbol | Unit (symbol) | Dimensions |
|---------------------------------|----------------|--|--------------------|
| Standing wave ratio | SWR | — | — |
| Surface charge density | ρ_S | coulomb/square meter (C/m ²) | $L^{-2}Q$ |
| Surface current density | \mathbf{J}_S | ampere/meter (A/m) | $L^{-1}T^{-1}Q$ |
| Susceptance | B | siemens (S) | $M^{-1}L^{-2}TQ^2$ |
| Time | t | second (s) | T |
| Transmission coefficient | τ | — | — |
| Unit normal vector | \mathbf{a}_n | — | — |
| Velocity | v | meter/second (m/s) | LT^{-1} |
| Velocity of light in free space | c | meter/second (m/s) | LT^{-1} |
| Voltage | V | volt (V) | $ML^2T^{-2}Q^{-1}$ |
| Volume | V | cubic meter (m ³) | L^3 |
| Volume charge density | ρ | coulomb/cubic meter (C/m ³) | $L^{-3}Q$ |
| Volume current density | \mathbf{J} | ampere/square meter (A/m ²) | $L^{-2}T^{-1}Q$ |
| Wavelength | λ | meter (m) | L |
| Work | W | joule (J) | ML^2T^{-2} |

Dimensions are a convenient means of checking the possible validity of a derived equation. The dimension of a given quantity can be expressed as some combination of a set of fundamental dimensions. These fundamental dimensions are mass (M), length (L), and time (T). In electromagnetics, it is the usual practice to consider the charge (Q), instead of the current, as the additional fundamental dimension. For the quantities listed in Table C.1, these four dimensions are sufficient. Thus, for example, the dimension of velocity is length (L) divided by time (T), that is, LT^{-1} ; the dimension of acceleration is length (L) divided by time squared (T^2), that is, LT^{-2} ; the dimension of force is mass (M) times acceleration (LT^{-2}), that is, MLT^{-2} ; the dimension of ampere is charge (Q) divided by time (T), that is, QT^{-1} ; and so on.

To illustrate the application of dimensions for checking the possible validity of a derived equation, let us consider the equation for the phase velocity of an electromagnetic wave in free space, given by

$$v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

We know that the dimension of v_p is LT^{-1} . Hence, we have to show that the dimension of $1/\sqrt{\mu_0\epsilon_0}$ is also LT^{-1} . To do this, we note from Coulomb's law that

$$\epsilon_0 = \frac{Q_1Q_2}{4\pi FR^2}$$

Hence, the dimension of ϵ_0 is $Q^2/[(MLT^{-2})(L^2)]$, or $M^{-1}L^{-3}T^2Q^2$. We note from Ampere's law of force applied to two infinitesimal current elements parallel to each other and normal to the line joining them that

$$\mu_0 = \frac{4\pi FR^2}{(I_1 dl_1)(I_2 dl_2)}$$

Hence, the dimension of μ_0 is $[(MLT^{-2})(L^2)]/(QT^{-1}L)^2$, or MLQ^{-2} . Now we obtain the dimension of $1/\sqrt{\mu_0\epsilon_0}$ as $1/\sqrt{(M^{-1}L^{-3}T^2Q^2)(MLQ^{-2})}$, or LT^{-1} , which is the same as the dimension of v_p . It should, however, be noted that the test for the equality of the dimensions of the two sides of a derived equation is not a sufficient test to establish the equality of the two sides, since any dimensionless constants associated with the equation may be in error.

It is not always necessary to refer to the table of dimensions for checking the possible validity of a derived equation. For example, let us assume that we have derived the expression for the characteristic impedance of a transmission line, that is, $\sqrt{\mathcal{L}/\mathcal{C}}$, and we wish to verify that $\sqrt{\mathcal{L}/\mathcal{C}}$ does indeed have the dimension of impedance. To do this, we write

$$\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{\omega \mathcal{L}l}{\omega \mathcal{C}l}} = \sqrt{\frac{\omega L}{\omega C}} = \sqrt{(\omega L)\left(\frac{1}{\omega C}\right)}$$

We now recognize from our knowledge of circuit theory that both ωL and $1/\omega C$, being the reactances of L and C , respectively, have the dimension of impedance. Hence, we conclude that $\sqrt{\mathcal{L}/\mathcal{C}}$ has the dimension of $\sqrt{(\text{impedance})^2}$, or impedance.