CHAPTER

Antenna Basics

9

In the preceding chapters, we studied the principles of propagation and transmission of electromagnetic waves. The remaining important topic pertinent to electromagnetic wave phenomena is radiation of electromagnetic waves. We have, in fact, touched on the principle of radiation of electromagnetic waves in Chapter 4 when we derived the electromagnetic field due to the infinite plane sheet of sinusoidally time-varying, spatially uniform current density. We learned that the current sheet gives rise to uniform plane waves *radiating* away from the sheet to either side of it. We pointed out at that time that the infinite plane current sheet is, however, an idealized, hypothetical source. With the experience gained thus far in our study of the elements of engineering electromagnetics, we are now in a position to learn the principles of radiation from physical antennas, which is our goal in this chapter.

We shall begin the chapter with the derivation of the electromagnetic field due to an elemental wire antenna, known as the *Hertzian dipole.* After studying the radiation characteristics of the Hertzian dipole, we shall consider the example of a half-wave dipole to illustrate the use of superposition to represent an arbitrary wire antenna as a series of Hertzian dipoles in order to determine its radiation fields. We shall also discuss the principles of arrays of physical antennas and the concept of image antennas to take into account ground effects. Finally, we shall briefly consider the receiving properties of antennas and learn of their reciprocity with the radiating properties.

9.1 HERTZIAN DIPOLE

The Hertzian dipole is an elemental antenna consisting of an infinitesimally long piece of wire carrying an alternating current $I(t)$, as shown in Figure 9.1. To maintain the current flow in the wire, we postulate two point charges $Q_1(t)$ and $Q_2(t)$ terminating the wire at its two ends, so that the law of conservation of charge is satisfied. Thus, if

$$I(t) = I_0 \cos \omega t \tag{9.1}$$

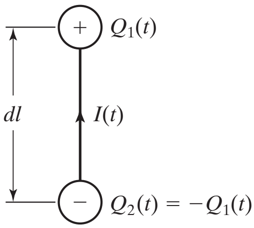


FIGURE 9.1
Hertzian dipole.

then

$$\frac{dQ_1}{dt} = I(t) = I_0 \cos \omega t \tag{9.2a}$$

$$\frac{dQ_2}{dt} = -I(t) = -I_0 \cos \omega t \tag{9.2b}$$

and

$$Q_1(t) = \frac{I_0}{\omega} \sin \omega t \tag{9.3a}$$

$$Q_2(t) = -\frac{I_0}{\omega} \sin \omega t = -Q_1(t) \tag{9.3b}$$

The time variations of I, Q_1, and Q_2, given by (9.1), (9.3a), and (9.3b), respectively, are illustrated by the curves and the series of sketches for the dipoles in Figure 9.2, corresponding to one complete period. The different sizes of the arrows associated with the dipoles denote the different strengths of the current, whereas the number of the plus or minus signs is indicative of the strength of the charges.

To determine the electromagnetic field due to the Hertzian dipole, we shall employ an intuitive approach based upon the knowledge gained in the previous chapters, as follows: From the application of what we have learned in Chapter 1, we can obtain the expressions for the electric and magnetic fields due to the point charges and the current element, respectively, associated with the Hertzian dipole, assuming that the fields follow exactly the time-variations of the charges and the current. These expressions do not, however, take into account the fact that time-varying electric and magnetic fields give rise to wave propagation. Hence, we shall extend them from considerations of our knowledge of wave propagation and then check if the resulting solutions satisfy Maxwell's equations. If they do not, we will then have to modify them so that they do satisfy Maxwell's equations and at the same time reduce to the originally derived expressions in the region where wave propagation effects are small, that is, at distances from the dipole that are small compared to a wavelength.

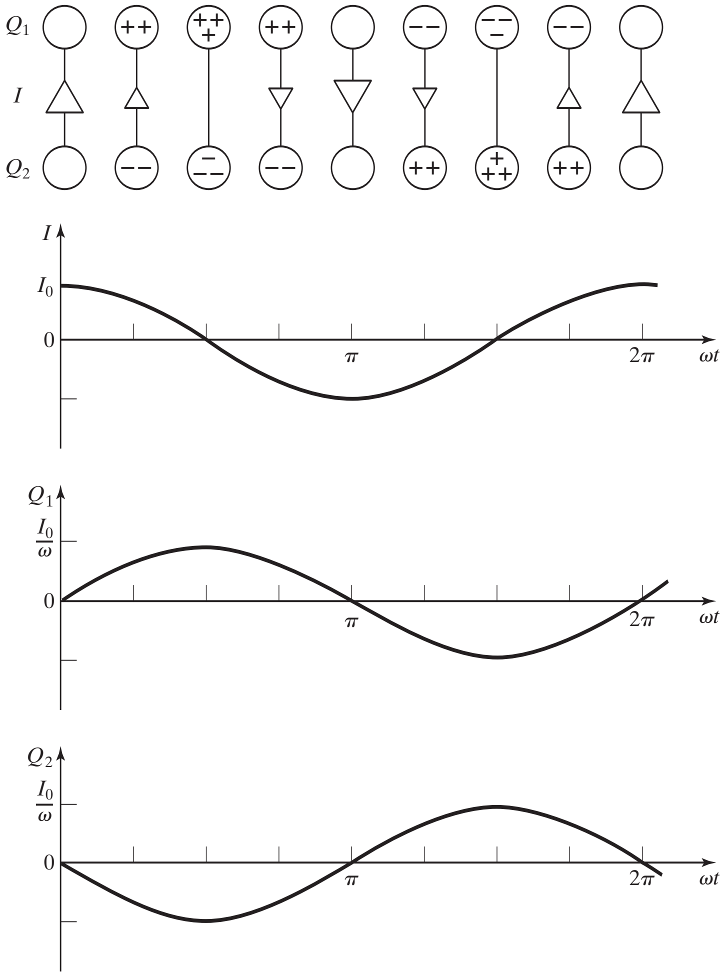


FIGURE 9.2

Time variations of charges and current associated with the Hertzian dipole.

To follow the approach outlined in the preceding paragraph, we locate the dipole at the origin with the current directed along the z-axis, as shown in Figure 9.3, and derive first the expressions for the fields by applying the simple laws learned in Sections 1.5 and 1.6. The symmetry associated with the problem is such that it is simpler to use a spherical coordinate system. Hence, if the reader is not already familiar with the spherical coordinate system, it is suggested that Appendix A be read at this stage. To review briefly, a point in the spherical coordinate system is defined by the intersection of a sphere centered at the origin, a cone having its apex at the origin and its surface symmetrical about the z-axis, and a plane containing the z-axis. Thus, the coordinates for a given point, say P, are r, the radial distance from the origin, θ, the angle which the radial line from the origin to the point makes with the z-axis, and ϕ, the angle which the line drawn from the origin to the projection of the point onto the xy-plane makes with the x-axis, as shown in Figure 9.3. A vector drawn at a given point is represented in terms of the unit vectors $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_\phi$ directed in the increasing r, θ, and ϕ directions, respectively, at that point. It is important to note that all three of these unit vectors are not uniform unlike the unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ in the Cartesian coordinate system.

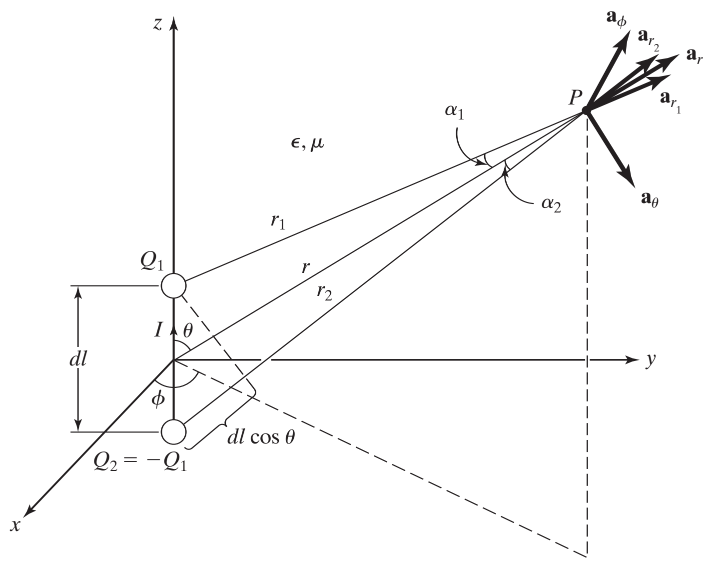


FIGURE 9.3

For the determination of the electromagnetic field due to the Hertzian dipole.

Now using the expression for the electric field due to a point charge given by (1.52), we can write the electric field at point P due to the arrangement of the two point charges Q_1 and $-Q_1$ in Figure 9.3 to be

$$\mathbf{E} = \frac{Q_1}{4\pi\epsilon r_1^2}\mathbf{a}_{r_1} - \frac{Q_1}{4\pi\epsilon r_2^2}\mathbf{a}_{r_2} \tag{9.4}$$

where r_1 and r_2 are the distances from Q_1 to P and $Q_2 (= -Q_1)$ to P, respectively, and $\mathbf{a}_{r_1}$ and $\mathbf{a}_{r_2}$ are unit vectors directed along the lines from Q_1 to P and Q_2 to P, respectively, as shown in Figure 9.3. Noting that

$$\mathbf{a}_{r_1} = \cos\alpha_1\,\mathbf{a}_r + \sin\alpha_1\,\mathbf{a}_\theta \tag{9.5a}$$

$$\mathbf{a}_{r_2} = \cos\alpha_2\,\mathbf{a}_r - \sin\alpha_2\,\mathbf{a}_\theta \tag{9.5b}$$

we obtain the r and θ components of the electric field at P to be

$$E_r = \frac{Q_1}{4\pi\epsilon}\left(\frac{\cos\alpha_1}{r_1^2} - \frac{\cos\alpha_2}{r_2^2}\right) \tag{9.6a}$$

$$E_\theta = \frac{Q_1}{4\pi\epsilon}\left(\frac{\sin\alpha_1}{r_1^2} + \frac{\sin\alpha_2}{r_2^2}\right) \tag{9.6b}$$

For infinitesimal value of the length dl of the current element, that is, for $dl \ll r$,

$$\begin{aligned}\left(\frac{\cos\alpha_1}{r_1^2} - \frac{\cos\alpha_2}{r_2^2}\right) &\approx \frac{1}{r_1^2} - \frac{1}{r_2^2} \\ &= \frac{(r_2 - r_1)(r_2 + r_1)}{r_1^2 r_2^2} \approx \frac{(dl\cos\theta)\,2r}{r^4} \\ &= \frac{2\,dl\cos\theta}{r^3}\end{aligned} \tag{9.7a}$$

and

$$\begin{aligned}\left(\frac{\sin\alpha_1}{r_1^2} + \frac{\sin\alpha_2}{r_2^2}\right) &\approx \frac{2\sin\alpha_1}{r^2} \\ &\approx \frac{dl\sin\theta}{r^3}\end{aligned} \tag{9.7b}$$

where we have also used the approximations that for $dl \ll r$, $(r_2 - r_1) \approx dl\cos\theta$ and $\sin\alpha_1 \approx [(dl/2)\sin\theta]/r$. These are, of course, exact in the limit that $dl \to 0$. Substituting (9.7a) and (9.7b) in (9.6a) and (9.6b), respectively, we obtain the electric field at point P due to the arrangement of the two point charges to be given by

$$\mathbf{E} = \frac{Q_1\,dl}{4\pi\epsilon r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) \tag{9.8}$$

Note that $Q_1\,dl$ is the electric dipole moment associated with the Hertzian dipole.

Using the Biot–Savart law given by (1.68), we can write the magnetic field at point P due to the infinitesimal current element in Figure 9.3 to be

$$\begin{aligned}\mathbf{H} = \frac{\mathbf{B}}{\mu} &= \frac{I\,dl\,\mathbf{a}_z \times \mathbf{a}_r}{4\pi r^2} \\ &= \frac{I\,dl\sin\theta}{4\pi r^2}\,\mathbf{a}_\phi\end{aligned} \tag{9.9}$$

To extend the expressions for **E** and **H** given by (9.8) and (9.9), respectively, we observe that when the charges and current vary with time, the fields also vary with time giving rise to wave propagation. The effect of a given time-variation of the source quantity is therefore felt at a point in space not instantaneously but only after a time lag. This time lag is equal to the time it takes for the wave to propagate from the source point to the observation point, that is, r/v_p, or $\beta r/\omega$, where $v_p(=1/\sqrt{\mu\epsilon})$ and $\beta(=\omega\sqrt{\mu\epsilon})$ are the phase velocity and the phase constant, respectively. Thus, for

$$Q_1 = \frac{I_0}{\omega}\sin \omega t \tag{9.10}$$

$$I = I_0 \cos \omega t \tag{9.11}$$

we would intuitively expect the fields at point P to be given by

$$\begin{aligned}\mathbf{E} &= \frac{[(I_0/\omega)\sin \omega(t - \beta r/\omega)]\,dl}{4\pi\epsilon r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) \\ &= \frac{I_0\,dl \sin(\omega t - \beta r)}{4\pi\epsilon\omega r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)\end{aligned} \tag{9.12a}$$

$$\begin{aligned}\mathbf{H} &= \frac{[I_0\cos \omega(t - \beta r/\omega)]\,dl}{4\pi r^2}\sin\theta\,\mathbf{a}_\phi \\ &= \frac{I_0\,dl \cos(\omega t - \beta r)}{4\pi r^2}\sin\theta\,\mathbf{a}_\phi\end{aligned} \tag{9.12b}$$

There is, however, one thing wrong with our intuitive expectation of the fields due to the Hertzian dipole! The fields do not satisfy Maxwell's curl equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu\frac{\partial \mathbf{H}}{\partial t} \tag{9.13a}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \epsilon\frac{\partial \mathbf{E}}{\partial t} \tag{9.13b}$$

(where we have set $\mathbf{J} = 0$ in view of the perfect dielectric medium). For example, let us try the curl equation for $\mathbf{H}$. First, we note from Appendix B that the expansion for the curl of a vector in spherical coordinates is

$$\begin{aligned}\nabla \times \mathbf{A} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r \\ &\quad + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\mathbf{a}_\theta \\ &\quad + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi\end{aligned} \tag{9.14}$$

Thus,

$$\begin{aligned}\nabla \times \mathbf{H} &= \frac{1}{r \sin\theta}\frac{\partial}{\partial\theta}\left[\frac{I_0\,dl\cos(\omega t-\beta r)}{4\pi r^2}\sin^2\theta\right]\mathbf{a}_r \\ &\quad -\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{I_0\,dl\cos(\omega t-\beta r)}{4\pi r}\sin\theta\right]\mathbf{a}_\theta \\ &= \frac{I_0\,dl\cos(\omega t-\beta r)}{4\pi r^3}(2\cos\theta\;\mathbf{a}_r+\sin\theta\;\mathbf{a}_\theta) \\ &\quad -\frac{\beta I_0\,dl\sin(\omega t-\beta r)}{4\pi r^2}\sin\theta\;\mathbf{a}_\theta \\ &= \epsilon\frac{\partial\mathbf{E}}{\partial t}-\frac{\beta I_0\,dl\sin(\omega t-\beta r)}{4\pi r^2}\sin\theta\;\mathbf{a}_\theta \\ &\neq \epsilon\frac{\partial\mathbf{E}}{\partial t}\end{aligned} \tag{9.15}$$

The reason behind the discrepancy associated with the expressions for the fields due to the Hertzian dipole can be understood by recalling that in Section 4.6 we learned from considerations of the Poynting vector that the fields far from a physical antenna vary inversely with the radial distance away from the antenna. The expressions we have derived do not contain inverse distance dependent terms and hence they are not complete, thereby causing the discrepancy. The complete field expressions must contain terms involving $1/r$ in addition to those in (9.12a) and (9.12b). Since for small r, $1/r \ll 1/r^2 \ll 1/r^3$, the addition of terms involving $1/r$ and containing $\sin\theta$ to (9.12a) and (9.12b) would still maintain the fields in the region close to the dipole to be predominantly the same as those given by (9.12a) and (9.12b), while making the $1/r$ terms predominant for large r, since for large r, $1/r \gg 1/r^2 \gg 1/r^3$.

Thus, let us modify the expression for **H** given by (9.12b) by adding a second term containing $1/r$ in the following manner:

$$\mathbf{H} = \frac{I_0\,dl\sin\theta}{4\pi}\left[\frac{\cos(\omega t-\beta r)}{r^2}+\frac{A\cos(\omega t-\beta r+\delta)}{r}\right]\mathbf{a}_\phi \tag{9.16}$$

where A and δ are constants to be determined. Then from Maxwell's curl equation for **H**, given by (9.13b), we have

$$\begin{aligned}\epsilon\frac{\partial\mathbf{E}}{\partial t} = \nabla\times\mathbf{H} &= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(H_\phi\sin\theta)\;\mathbf{a}_r-\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)\;\mathbf{a}_\theta \\ &= \frac{2I_0\,dl\cos\theta}{4\pi}\left[\frac{\cos(\omega t-\beta r)}{r^3}+\frac{A\cos(\omega t-\beta r+\delta)}{r^2}\right]\mathbf{a}_r \\ &\quad +\frac{I_0\,dl\sin\theta}{4\pi}\left[\frac{\cos(\omega t-\beta r)}{r^3}-\frac{\beta\sin(\omega t-\beta r)}{r^2}\right. \\ &\quad \left.-\frac{A\beta\sin(\omega t-\beta r+\delta)}{r}\right]\mathbf{a}_\theta\end{aligned} \tag{9.17}$$

$$\mathbf{E} = \frac{2I_0\, dl \cos\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{A\sin(\omega t - \beta r + \delta)}{r^2}\right]\mathbf{a}_r$$
$$+ \frac{I_0\, dl \sin\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2}\right.$$
$$\left. + \frac{A\beta\cos(\omega t - \beta r + \delta)}{r}\right]\mathbf{a}_\theta \tag{9.18}$$

Now, from Maxwell's curl equation for **E** given by (9.13a), we have

$$\mu\frac{\partial\mathbf{H}}{\partial t} = -\nabla\times\mathbf{E} = -\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta}\right]\mathbf{a}_\phi$$
$$= \frac{I_0 dl \sin\theta}{4\pi\epsilon\omega}\left[\frac{2\beta\cos(\omega t - \beta r)}{r^3} - \frac{2A\sin(\omega t - \beta r + \delta)}{r^3}\right.$$
$$\left. - \frac{\beta^2\sin(\omega t - \beta r)}{r^2} - \frac{A\beta^2\sin(\omega t - \beta r + \delta)}{r}\right]\mathbf{a}_\phi \tag{9.19}$$

$$\mathbf{H} = \frac{I_0\, dl \sin\theta}{4\pi}\left[\frac{2\sin(\omega t - \beta r)}{\beta r^3} + \frac{2A\cos(\omega t - \beta r + \delta)}{\beta^2 r^3}\right.$$
$$\left. + \frac{\cos(\omega t - \beta r)}{r^2} + \frac{A\cos(\omega t - \beta r + \delta)}{r}\right]\mathbf{a}_\phi \tag{9.20}$$

We, however, have to rule out the $1/r^3$ terms in (9.20), since for small r these terms are more predominant than the $1/r^2$ dependence required by (9.12b). Equation (9.20) will then also be consistent with (9.16) from which we derived (9.18) and then (9.20). Thus, we set

$$\frac{2\sin(\omega t - \beta r)}{\beta r^3} + \frac{2A\cos(\omega t - \beta r + \delta)}{\beta^2 r^3} = 0 \tag{9.21}$$

which gives us

$$\delta = \frac{\pi}{2} \tag{9.22a}$$
$$A = \beta \tag{9.22b}$$

Substituting (9.22a) and (9.22b) in (9.18) and (9.20), we then have

$$\mathbf{E} = \frac{2I_0\, dl \cos\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2}\right]\mathbf{a}_r$$
$$+ \frac{I_0\, dl \sin\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2}\right.$$
$$\left. - \frac{\beta^2\sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\theta \tag{9.23a}$$

$$\mathbf{H} = \frac{I_0\,dl \sin\theta}{4\pi}\left[\frac{\cos(\omega t - \beta r)}{r^2} - \frac{\beta \sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\phi \tag{9.23b}$$

These expressions for **E** and **H** satisfy both of Maxwell's curl equations, reduce to (9.12a) and (9.12b), respectively, for small r ($\beta r \ll 1$), and they vary inversely with r for large r ($\beta r \gg 1$). They represent the complete electromagnetic field due to the Hertzian dipole.

9.2 RADIATION RESISTANCE AND DIRECTIVITY

In the previous section, we derived the expressions for the complete electromagnetic field due to the Hertzian dipole. These expressions look very complicated. Fortunately, it is seldom necessary to work with the complete field expressions because one is often interested in the field far from the dipole, which is governed predominantly by the terms involving $1/r$. We, however, had to derive the complete field in order to obtain the amplitude and phase of these $1/r$ terms relative to the amplitude and phase of the current in the Hertzian dipole, since these terms alone do not satisfy Maxwell's equations. Furthermore, by going through the exercise, we learned how to solve a difficult problem through intuitive extension and reasoning based on previously gained knowledge.

Thus from (9.23a) and (9.23b), we find that for a Hertzian dipole of length dl oriented along the z-axis and carrying current

$$I = I_0 \cos \omega t \tag{9.24}$$

the electric and magnetic fields at values of r far from the dipole are given by

$$\begin{aligned}\mathbf{E} &= -\frac{\beta^2 I_0\,dl \sin\theta}{4\pi\epsilon\omega r}\sin(\omega t - \beta r)\,\mathbf{a}_\theta \\ &= -\frac{\eta\beta I_0\,dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\theta\end{aligned} \tag{9.25a}$$

$$\mathbf{H} = -\frac{\beta I_0\,dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\phi \tag{9.25b}$$

These fields are known as the *radiation fields*, since they are the components of the total fields that contribute to the time-average radiated power away from the dipole (see Problem 9.6). Before we discuss the nature of these fields, let us find out quantitatively what we mean by *far from the dipole.* To do this, we look at the expression for the complete magnetic field given by (9.23b) and note that the ratio of the amplitudes of the $1/r^2$ and $1/r$ terms is equal to $1/\beta r$. Hence for $\beta r \gg 1$, or $r \gg \lambda/2\pi$, the $1/r^2$ term is negligible compared to the $1/r$ term. Thus, even at a distance of a few wavelengths from the dipole, the fields are predominantly radiation fields.

Returning now to the expressions for the radiation fields given by (9.25a) and (9.25b), we note that at any given point, (a) the electric field (E_θ), the magnetic field (H_ϕ), and the direction of propagation (r) are mutually perpendicular, and (b) the ratio of E_θ to H_ϕ is equal to η, the intrinsic impedance of the medium, which are characteristic of uniform plane waves. The phase of the field, however, is uniform over the surfaces r = constant, that is, spherical surfaces centered at the dipole, whereas the amplitude of the field is uniform over surfaces $(\sin\theta)/r$ = constant. Hence, the fields are only locally uniform plane waves, that is, over any infinitesimal area normal to the r-direction at a given point.

The Poynting vector due to the radiation fields is given by

$$\begin{aligned}\mathbf{P} &= \mathbf{E} \times \mathbf{H}\\ &= E_\theta \mathbf{a}_\theta \times H_\phi \mathbf{a}_\phi = E_\theta H_\phi \mathbf{a}_r\\ &= \frac{\eta\beta^2 I_0^2 (dl)^2 \sin^2\theta}{16\pi^2 r^2} \sin^2(\omega t - \beta r)\, \mathbf{a}_r \end{aligned} \tag{9.26}$$

By evaluating the surface integral of the Poynting vector over any surface enclosing the dipole, we can find the power flow out of that surface, that is, the power *radiated* by the dipole. For convenience in evaluating the surface integral, we choose the spherical surface of radius r and centered at the dipole, as shown in Figure 9.4. Thus, noting that the differential surface area on the spherical surface is $(r\,d\theta)(r\sin\theta\,d\phi)\mathbf{a}_r$ or $r^2\sin\theta\,d\theta\,d\phi\,\mathbf{a}_r$, we obtain the instantaneous power radiated to be

$$\begin{aligned}P_{\text{rad}} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \mathbf{P}\cdot r^2\sin\theta\,d\theta\,d\phi\,\mathbf{a}_r\\ &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{\eta\beta^2 I_0^2 (dl)^2\sin^3\theta}{16\pi^2}\sin^2(\omega t-\beta r)\,d\theta\,d\phi\\ &= \frac{\eta\beta^2 I_0^2 (dl)^2}{8\pi}\sin^2(\omega t-\beta r)\int_{\theta=0}^{\pi}\sin^3\theta\,d\theta\\ &= \frac{\eta\beta^2 I_0^2 (dl)^2}{6\pi}\sin^2(\omega t-\beta r)\\ &= \frac{2\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2\sin^2(\omega t-\beta r)\end{aligned} \tag{9.27}$$

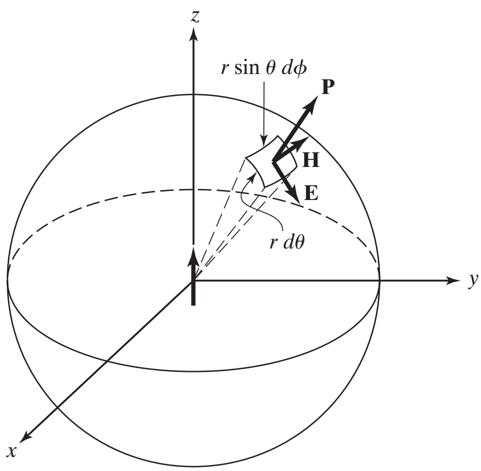


FIGURE 9.4

For computing the power radiated by the Hertzian dipole.

The time-average power radiated by the dipole, that is, the average of P_{rad} over one period of the current variation, is

$$\begin{aligned}\langle P_{\text{rad}} \rangle &= \frac{2\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2 \langle \sin^2(\omega t - \beta r)\rangle \\ &= \frac{\pi\eta I_0^2}{3}\left(\frac{dl}{\lambda}\right)^2 \\ &= \frac{1}{2} I_0^2 \left[\frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2\right] \end{aligned} \tag{9.28}$$

We now define a quantity known as the *radiation resistance* of the antenna, denoted by the symbol R_{rad}, as the value of a fictitious resistor that dissipates the same amount of time-average power as that radiated by the antenna when a current of the same peak amplitude as that in the antenna is passed through it. Recalling that the average power dissipated in a resistor R when a current $I_0 \cos \omega t$ is passed through it is $\frac{1}{2}I_0^2 R$, we note from (9.28) that the radiation resistance of the Hertzian dipole is

$$R_{\text{rad}} = \frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2 \ \Omega \tag{9.29}$$

For free space, $\eta = \eta_0 = 120\pi\ \Omega$, and

$$R_{\text{rad}} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \ \Omega \tag{9.30}$$

As a numerical example, for (dl/λ) equal to 0.01, $R_{\text{rad}} = 80\pi^2(0.01)^2 = 0.08\ \Omega$. Thus, for a current of peak amplitude 1 A, the time-average radiated power is equal to 0.04 W. This indicates that a Hertzian dipole of length 0.01λ is not a very effective radiator.

We note from (9.29) that the radiation resistance and hence the radiated power are proportional to the square of the electrical length, that is, the physical length expressed in terms of wavelength, of the dipole. The result given by (9.29) is, however, valid only for small values of dl/λ, since if dl/λ is not small, the amplitude of the current along the antenna can no longer be uniform and its variation must be taken into account in deriving the radiation fields and hence the radiation resistance. We shall do this in the following section for a half-wave dipole, that is, for a dipole of length equal to $\lambda/2$.

Let us now examine the directional characteristics of the radiation from the Hertzian dipole. We note from (9.25a) and (9.25b) that, for a constant r, the amplitude of the fields is proportional to $\sin\theta$. Similarly, we note from (9.26) that, for a constant r, the power density is proportional to $\sin^2\theta$. Thus, an observer wandering on the surface of an imaginary sphere centered at the dipole views different amplitudes of the fields and of the power density at different points on the surface. The situation is illustrated in Figure 9.5(a) for the power density by attaching to different points on the spherical surface vectors having lengths proportional to the Poynting vectors at those points. It can be seen that the power density is largest for $\theta = \pi/2$, that is, in the plane normal to

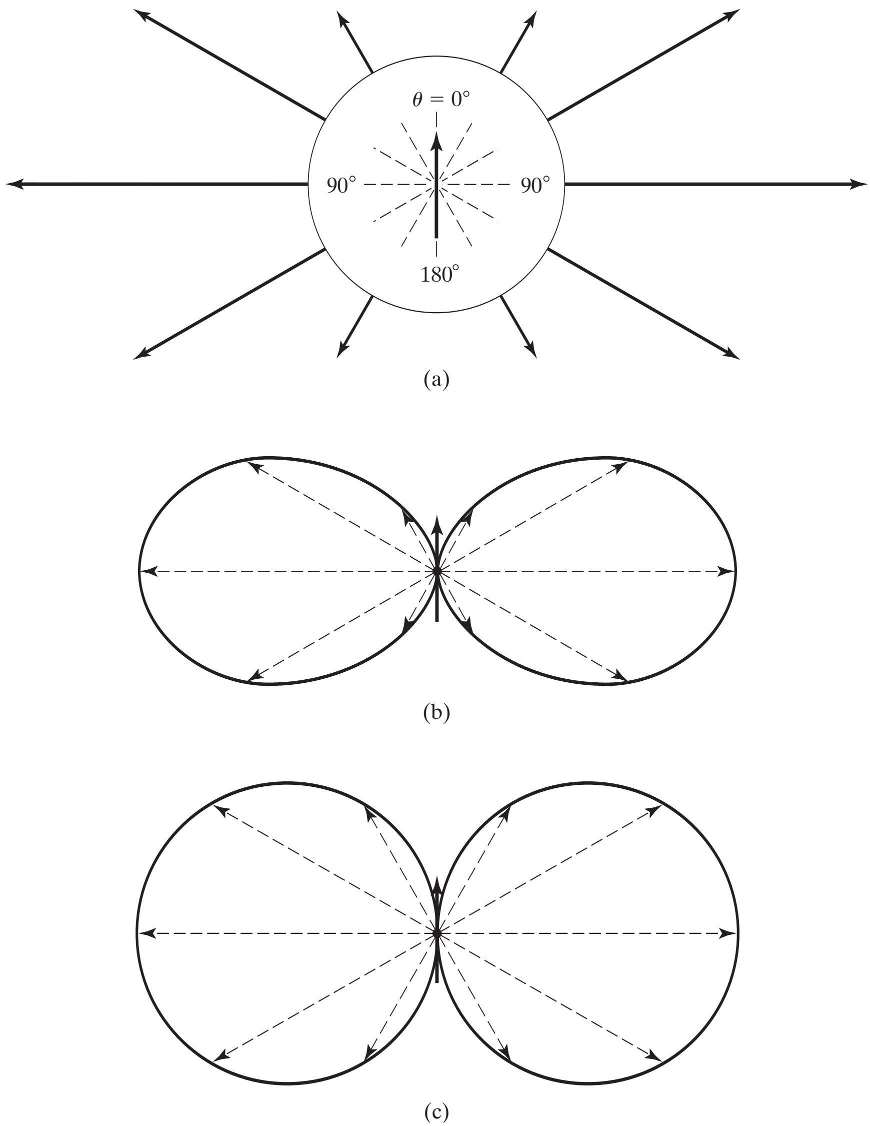


FIGURE 9.5

The directional characteristics of radiation from the Hertzian dipole.

the axis of the dipole, and decreases continuously toward the axis of the dipole, becoming zero along the axis.

It is customary to depict the radiation characteristic by means of a *radiation pattern*, as shown in Figure 9.5(b), which can be imagined to be obtained by shrinking the radius of the spherical surface in Figure 9.5(a) to zero with the Poynting vectors attached to it and then joining the tips of the Poynting vectors. Thus, the distance from the dipole point to a point on the radiation pattern is proportional to the power density in the direction of that point. Similarly, the radiation pattern for the fields can be drawn as shown in Figure 9.5(c), based upon the $\sin\theta$ dependence of the fields. In view of the independence of the fields from ϕ, the patterns of Figure 9.5(b)–(c) are valid for any plane containing the axis of the dipole. In fact, the three-dimensional radiation patterns can be imagined to be the figures obtained by revolving these patterns about the dipole axis. For a general case, the radiation may also depend on ϕ, and hence it will be necessary to draw a radiation pattern for the $\theta = \pi/2$ plane. Here, this pattern is merely a circle centered at the dipole.

We now define a parameter known as the *directivity* of the antenna, denoted by the symbol D, as the ratio of the maximum power density radiated by the antenna to the average power density. To elaborate on the definition of D, imagine that we take the power radiated by the antenna and distribute it equally in all directions by shortening some of the vectors in Figure 9.5(a) and lengthening the others so that they all have equal lengths. The pattern then becomes nondirectional and the power density, which is the same in all directions, will be less than the maximum power density of the original pattern. Obviously, the more directional the radiation pattern of an antenna is, the greater is the directivity.

From (9.26), we obtain the maximum power density radiated by the Hertzian dipole to be

$$\begin{aligned} [P_r]_{\max} &= \frac{\eta\beta^2 I_0^2 (dl)^2 [\sin^2\theta]_{\max}}{16\pi^2 r^2}\sin^2(\omega t - \beta r) \\ &= \frac{\eta\beta^2 I_0^2 (dl)^2}{16\pi^2 r^2}\sin^2(\omega t - \beta r) \end{aligned} \tag{9.31}$$

By dividing the radiated power given by (9.27) by the surface area $4\pi r^2$ of the sphere of radius r, we obtain the average power density to be

$$[P_r]_{\text{av}} = \frac{P_{\text{rad}}}{4\pi r^2} = \frac{\eta\beta^2 I_0^2\,(dl)^2}{24\pi^2 r^2}\sin^2(\omega t - \beta r) \tag{9.32}$$

Thus, the directivity of the Hertzian dipole is given by

$$D = \frac{[P_r]_{\max}}{[P_r]_{\text{av}}} = 1.5 \tag{9.33}$$

To generalize the computation of directivity for an arbitrary radiation pattern, let us consider

$$P_r = \frac{P_0\sin^2(\omega t - \beta r)}{r^2} f(\theta, \phi) \tag{9.34}$$

where P_0 is a constant, and $f(\theta, \phi)$ is the power density pattern. Then

$$[P_r]_{\max} = \frac{P_0 \sin^2(\omega t - \beta r)}{r^2}[f(\theta, \phi)]_{\max}$$

$$\begin{aligned}[P_r]_{\text{av}} &= \frac{P_{\text{rad}}}{4\pi r^2} \\ &= \frac{1}{4\pi r^2}\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi}\frac{P_0 \sin^2(\omega t - \beta r)}{r^2} f(\theta, \phi)\, \mathbf{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r \\ &= \frac{P_0 \sin^2(\omega t - \beta r)}{4\pi r^2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} f(\theta, \phi) \sin\theta\, d\theta\, d\phi\end{aligned}$$

$$D = 4\pi \frac{[f(\theta, \phi)]_{\max}}{\displaystyle\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} f(\theta, \phi) \sin\theta\, d\theta\, d\phi} \tag{9.35}$$

Example 9.1

Let us compute the directivity corresponding to the power density pattern function $f(\theta, \phi) = \sin^2\theta \cos^2\theta$.

From (9.35),

$$\begin{aligned}D &= 4\pi \frac{[\sin^2\theta\cos^2\theta]_{\max}}{\displaystyle\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\sin^3\theta\cos^2\theta\, d\theta\, d\phi} \\ &= 4\pi\frac{\left[\frac{1}{4}\sin^2 2\theta\right]_{\max}}{\displaystyle 2\pi\int_{\theta=0}^{\pi}(\sin^3\theta - \sin^5\theta)\, d\theta} \\ &= \frac{1}{2}\frac{1}{(4/3) - (16/15)} \\ &= 1\frac{7}{8}\end{aligned}$$

The ratio of the power density radiated by the antenna as a function of direction to the average power density is given by $Df(\theta, \phi)$. This quantity is known as the *directive gain of the antenna*. Another useful parameter is the power gain of the antenna, which takes into account the ohmic power losses in the antenna. It is denoted by the symbol G and is proportional to the directive gain, the proportionality factor being the power efficiency of the antenna, which is the ratio of the power radiated by the antenna to the power supplied to it by the source of excitation.

9.3 HALF-WAVE DIPOLE

In the previous section, we found the radiation fields due to a Hertzian dipole, which is an elemental antenna of infinitesimal length. If we now have an antenna of any length having a specified current distribution, we can divide it into a series of Hertzian dipoles and by applying superposition can find the radiation fields for that antenna. We shall illustrate this procedure in this section by considering the half-wave dipole, which is a commonly used form of antenna.

The half-wave dipole is a center-fed, straight wire antenna of length L equal to $\lambda/2$ and having the current distribution

$$I(z) = I_0 \cos \frac{\pi z}{L} \cos \omega t \qquad \text{for } -\frac{L}{2} < z < \frac{L}{2} \tag{9.36}$$

where the dipole is assumed to be oriented along the z-axis with its center at the origin, as shown in Figure 9.6(a). As can be seen from Figure 9.6(a), the amplitude of the current distribution varies cosinusoidally along the antenna with zeros at the ends and maximum at the center. To see how this distribution comes about, the half-wave dipole may be imagined to be the evolution of an open-circuited transmission line with the conductors folded perpendicularly to the line at points $\lambda/4$ from the end of the line. The current standing wave pattern for an open-circuited line is shown in Figure 9.6(b). It consists of zero current at the open circuit and maximum current at $\lambda/4$ from the open circuit, that is, at points a and a'. Hence, it can be seen that when the conductors are folded perpendicularly to the line at a and a', the half-wave dipole shown in Figure 9.6(a) results.

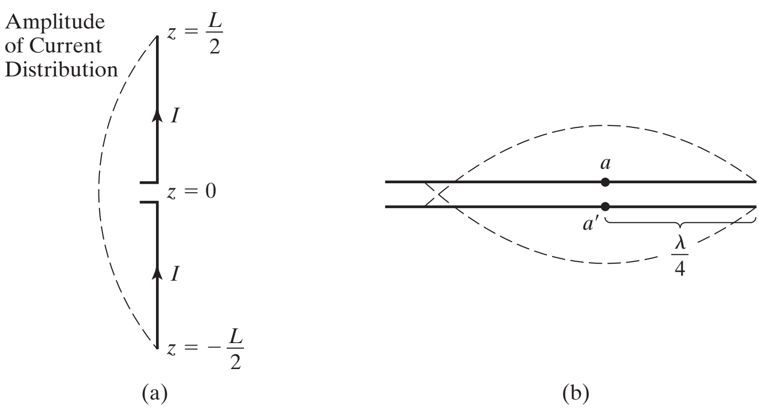


FIGURE 9.6

(a) Half-wave dipole. (b) Open-circuited transmission line for illustrating the evolution of the half-wave dipole.

Now, to find the radiation field due to the half-wave dipole, we divide it into a number of Hertzian dipoles, each of length dz', as shown in Figure 9.7. If we consider one of these dipoles situated at distance z' from the origin, then from (9.36) the current

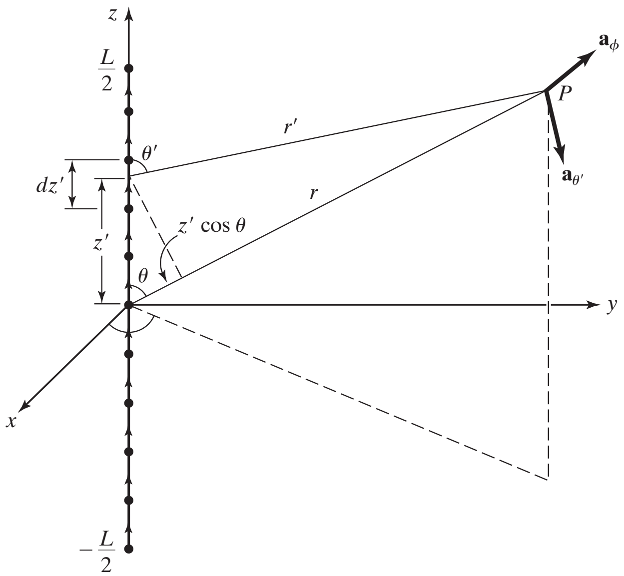


FIGURE 9.7

For the determination of the radiation field due to the half-wave dipole.

in this dipole is $I_0 \cos(\pi z'/L) \cos \omega t$. From (9.25a) and (9.25b), the radiation fields due to this dipole at point P situated at distance r' from it are given by

$$d\mathbf{E} = -\frac{\eta\beta I_0 \cos(\pi z'/L)\, dz' \sin\theta'}{4\pi r'} \sin(\omega t - \beta r')\, \mathbf{a}_{\theta'} \tag{9.37a}$$

$$d\mathbf{H} = -\frac{\beta I_0 \cos(\pi z'/L)\, dz' \sin\theta'}{4\pi r'} \sin(\omega t - \beta r')\, \mathbf{a}_{\phi} \tag{9.37b}$$

where θ' is the angle between the z-axis and the line from the current element to the point P and $\mathbf{a}_{\theta'}$ is the unit vector perpendicular to that line, as shown in Figure 9.7. The fields due to the entire current distribution of the half-wave dipole are then given by

$$\begin{aligned}\mathbf{E} &= \int_{z'=-L/2}^{L/2} d\mathbf{E} \\ &= -\int_{z'=-L/2}^{L/2} \frac{\eta\beta I_0 \cos(\pi z'/L) \sin\theta' dz'}{4\pi r'} \sin(\omega t - \beta r')\, \mathbf{a}_{\theta'}\end{aligned} \tag{9.38a}$$

$$\begin{aligned}\mathbf{H} &= \int_{z'=-L/2}^{L/2} d\mathbf{H} \\ &= -\int_{z'=-L/2}^{L/2} \frac{\beta I_0 \cos(\pi z'/L) \sin\theta' dz'}{4\pi r'} \sin(\omega t - \beta r')\, \mathbf{a}_{\phi}\end{aligned} \tag{9.38b}$$

where r', θ', and $\mathbf{a}_{\theta'}$ are functions of z'.

For radiation fields, r' is at least equal to several wavelengths and hence $\gg L$. We can therefore set $\mathbf{a}_{\theta'} \approx \mathbf{a}_\theta$ and $\theta' \approx \theta$, since they do not vary significantly for $-L/2 < z' < L/2$. We can also set $r' \approx r$ in the amplitude factors for the same reason, but for r' in the phase factors we substitute $r - z' \cos\theta$, since $\sin(\omega t - \beta r') = \sin(\omega t - \pi r'/L)$ can vary appreciably over the range $-L/2 < z' < L/2$. Thus, we have

$$\mathbf{E} = E_\theta \mathbf{a}_\theta$$

where

$$\begin{aligned} E_\theta &= -\int_{z'=-L/2}^{L/2} \frac{\eta \beta I_0 \cos(\pi z'/L) \sin\theta}{4\pi r} \sin(\omega t - \beta r + \beta z' \cos\theta)\, dz' \\ &= -\frac{\eta(\pi/L) I_0 \sin\theta}{4\pi r} \int_{z'=-L/2}^{L/2} \cos\frac{\pi z'}{L} \sin\left(\omega t - \frac{\pi}{L} r + \frac{\pi}{L} z' \cos\theta\right) dz' \\ &= -\frac{\eta I_0}{2\pi r} \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \sin\left(\omega t - \frac{\pi}{L} r\right) \end{aligned} \quad (9.39a)$$

Similarly,

$$\mathbf{H} = H_\phi \mathbf{a}_\phi$$

where

$$H_\phi = -\frac{I_0}{2\pi r} \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \sin\left(\omega t - \frac{\pi}{L} r\right) \quad (9.39b)$$

The Poynting vector due to the radiation fields of the half-wave dipole is given by

$$\begin{aligned} \mathbf{P} &= \mathbf{E} \times \mathbf{H} = E_\theta H_\phi \mathbf{a}_r \\ &= \frac{\eta I_0^2}{4\pi^2 r^2} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin^2\theta} \sin^2\left(\omega t - \frac{\pi}{L} r\right) \mathbf{a}_r \end{aligned} \quad (9.40)$$

The power radiated by the half-wave dipole is given by

$$\begin{aligned} P_{\text{rad}} &= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \mathbf{P} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r \\ &= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\eta I_0^2}{4\pi^2} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta} \sin^2\left(\omega t - \frac{\pi}{L} r\right) d\theta \, d\phi \\ &= \frac{\eta I_0^2}{\pi} \sin^2\left(\omega t - \frac{\pi}{L} r\right) \int_{\theta=0}^{\pi/2} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta} d\theta \\ &= \frac{0.609 \eta I_0^2}{\pi} \sin^2\left(\omega t - \frac{\pi}{L} r\right) \end{aligned} \quad (9.41)$$

The time-average radiated power is

$$\langle P_{\text{rad}} \rangle = \frac{0.609\eta I_0^2}{\pi} \left\langle \sin^2\left(\omega t - \frac{\pi}{L} r\right) \right\rangle$$
$$= \frac{1}{2} I_0^2 \left(\frac{0.609\eta}{\pi} \right) \tag{9.42}$$

Thus, the radiation resistance of the half-wave dipole is

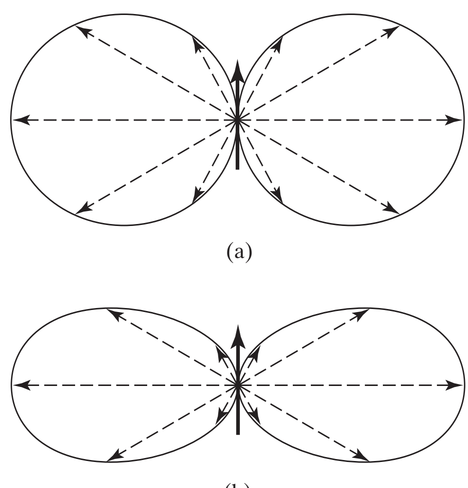
$$R_{\text{rad}} = \frac{0.609\eta}{\pi}\ \Omega \tag{9.43}$$

For free space, $\eta = \eta_0 = 120\pi\ \Omega$, and

$$R_{\text{rad}} = 0.609 \times 120 = 73\ \Omega \tag{9.44}$$

Turning our attention now to the directional characteristics of the half-wave dipole, we note from (9.39a) and (9.39b) that the radiation pattern for the fields is $\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right] \Big/ \sin\theta$ whereas for the power density, it is $\left[\cos^2\left(\frac{\pi}{2}\cos\theta\right)\right] \Big/ \sin^2\theta$. These patterns, which are sketched in Figure 9.8(a)–(b), are slightly more directional than the corresponding patterns for the Hertzian dipole. To find the directivity of the half-wave dipole, we note from (9.40) that the maximum power density is

$$[P_r]_{\max} = \frac{\eta I_0^2}{4\pi^2 r^2} \left\{ \frac{\cos^2[(\pi/2)\cos\theta]}{\sin^2\theta} \right\}_{\max} \sin^2\left(\omega t - \frac{\pi}{L} r\right)$$
$$= \frac{\eta I_0^2}{4\pi^2 r^2} \sin^2\left(\omega t - \frac{\pi}{L} r\right) \tag{9.45}$$



(a)

(b)

FIGURE 9.8

Radiation patterns for (a) the fields and (b) the power density due to the half-wave dipole.

The average power density obtained by dividing P_{rad} by $4\pi r^2$ is

$$[P_r]_{\text{av}} = \frac{0.609\eta I_0^2}{4\pi^2 r^2} \sin^2\left(\omega t - \frac{\pi}{L} r\right) \tag{9.46}$$

Thus, the directivity of the half-wave dipole is given by

$$D = \frac{[P_r]_{\text{max}}}{[P_r]_{\text{av}}} = \frac{1}{0.609} = 1.642 \tag{9.47}$$

9.4 ANTENNA ARRAYS

In Section 4.5, we illustrated the principle of an antenna array by considering an array of two parallel, infinite plane current sheets of uniform densities. We learned that by appropriately choosing the spacing between the current sheets and the amplitudes and phases of the current densities, a desired radiation characteristic can be obtained. The infinite plane current sheet is, however, a hypothetical antenna for which the fields are truly uniform plane waves propagating in the one dimension normal to the sheet. Now that we have gained some knowledge of physical antennas, in this section we shall consider arrays of such antennas.

The simplest array we can consider consists of two Hertzian dipoles, oriented parallel to the z-axis and situated at points on the x-axis on either side of and equidistant from the origin, as shown in Figure 9.9. We shall consider the amplitudes of the currents in the two dipoles to be equal, but we shall allow a phase difference α between them. Thus, if $I_1(t)$ and $I_2(t)$ are the currents in the dipoles situated at $(d/2, 0, 0)$ and $(-d/2, 0, 0)$, respectively, then

$$I_1 = I_0 \cos\left(\omega t + \frac{\alpha}{2}\right) \tag{9.48a}$$

$$I_2 = I_0 \cos\left(\omega t - \frac{\alpha}{2}\right) \tag{9.48b}$$

For simplicity, we shall consider a point P in the xz-plane and compute the field at that point due to the array of the two dipoles. To do this, we note from (9.25a) that the electric field intensities at the point P due to the individual dipoles are given by

$$\mathbf{E}_1 = -\frac{\eta\beta I_0 dl \sin\theta_1}{4\pi r_1} \sin\left(\omega t - \beta r_1 + \frac{\alpha}{2}\right)\mathbf{a}_{\theta_1} \tag{9.49a}$$

$$\mathbf{E}_2 = -\frac{\eta\beta I_0 dl \sin\theta_2}{4\pi r_2} \sin\left(\omega t - \beta r_2 - \frac{\alpha}{2}\right)\mathbf{a}_{\theta_2} \tag{9.49b}$$

where $\theta_1, \theta_2, r_1, r_2, \mathbf{a}_{\theta_1}$, and $\mathbf{a}_{\theta_2}$ are as shown in Figure 9.9.

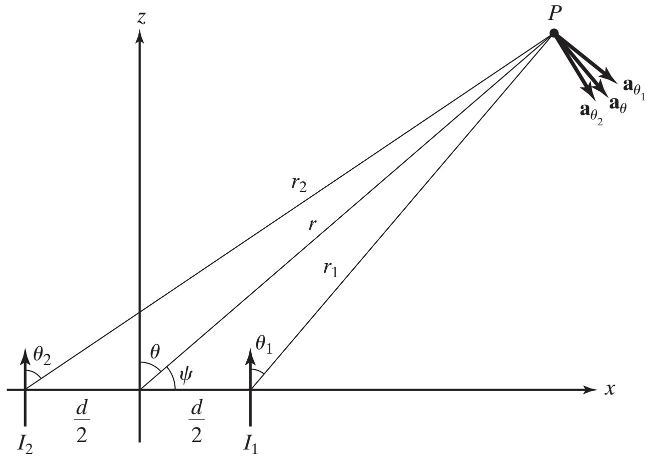


FIGURE 9.9

For computing the radiation field due to an array of two Hertzian dipoles.

For $r \gg d$, that is, for points far from the array, which is the region of interest, we can set $\theta_1 \approx \theta_2 \approx \theta$ and $\mathbf{a}_{\theta_1} \approx \mathbf{a}_{\theta_2} \approx \mathbf{a}_\theta$. Also, we can set $r_1 \approx r_2 \approx r$ in the amplitude factors, but for r_1 and r_2 in the phase factors, we substitute

$$r_1 \approx r - \frac{d}{2}\cos\psi \tag{9.50a}$$

$$r_2 \approx r + \frac{d}{2}\cos\psi \tag{9.50b}$$

where ψ is the angle made by the line form the origin to P with the axis of the array, that is, the x-axis, as shown in Figure 9.9. Thus we obtain the resultant field to be

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= -\frac{\eta\beta I_0\,dl\sin\theta}{4\pi r}\left[\sin\left(\omega t - \beta r + \frac{\beta d}{2}\cos\psi + \frac{\alpha}{2}\right)\right. \\ &\quad\left. + \sin\left(\omega t - \beta r - \frac{\beta d}{2}\cos\psi - \frac{\alpha}{2}\right)\right]\mathbf{a}_\theta \\ &= -\frac{2\eta\beta I_0\,dl\sin\theta}{4\pi r}\cos\left(\frac{\beta d\cos\psi + \alpha}{2}\right)\sin(\omega t - \beta r)\,\mathbf{a}_\theta \end{aligned} \tag{9.51}$$

Comparing (9.51) with the expression for the electric field at P due to a single dipole situated at the origin, we note that the resultant field of the array is simply equal to the single dipole field multiplied by the factor $2\cos\left(\frac{\beta d\cos\psi + \alpha}{2}\right)$, known as the *array factor*. Thus the radiation pattern of the resultant field is given by the product of $\sin\theta$, which is the radiation pattern of the single dipole field, and $\left|\cos\left(\frac{\beta d\cos\psi + \alpha}{2}\right)\right|$, which is the radiation pattern of the array if the antennas were isotropic. We shall call these three patterns the *resultant pattern*, the *unit pattern*, and the *group pattern*, respectively. It is apparent that the group pattern is independent of the nature of the individual antennas as long as they have the same spacing and carry currents having the same relative amplitudes and phase differences. It can also be seen that the group pattern is the same in any plane containing the axis of the array. In other words, the three-dimensional group pattern is simply the pattern obtained by revolving the group pattern in the xz-plane about the x-axis, that is, the axis of the array.

Example 9.2

For the array of two antennas carrying currents having equal amplitudes, let us consider several pairs of d and α and investigate the group patterns.

Case 1: $d = \lambda/2, \alpha = 0$. The group pattern is

$$\left|\cos\left(\frac{\beta\lambda}{4}\cos\psi\right)\right| = \cos\left(\frac{\pi}{2}\cos\psi\right)$$

This is shown sketched in Figure 9.10(a). It has maxima perpendicular to the axis of the array and nulls along the axis of the array. Such a pattern is known as a *broadside pattern*.

Case 2: $d = \lambda/2, \alpha = \pi$. The group pattern is

$$\left|\cos\left(\frac{\beta\lambda}{4}\cos\psi + \frac{\pi}{2}\right)\right| = \left|\sin\left(\frac{\pi}{2}\cos\psi\right)\right|$$

This is shown sketched in Figure 9.10(b). It has maxima along the axis of the array and nulls perpendicular to the axis of the array. Such a pattern is known as an *endfire pattern*.

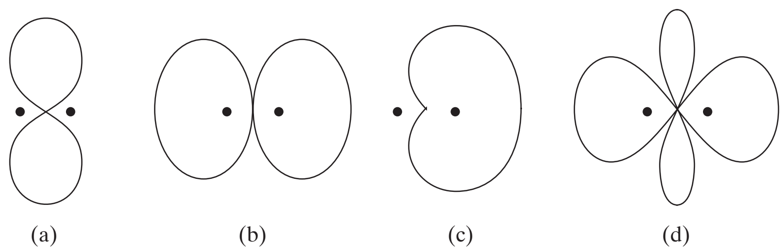


FIGURE 9.10

Group patterns for an array of two antennas carrying currents of equal amplitude for (a) $d = \lambda/2, \alpha = 0$, (b) $d = \lambda/2, \alpha = \pi$, (c) $d = \lambda/4, \alpha = -\pi/2$, and (d) $d = \lambda, \alpha = 0$.

Case 3: $d = \lambda/4, \alpha = -\pi/2$. The group pattern is

$$\left|\cos\left(\frac{\beta\lambda}{8}\cos\psi - \frac{\pi}{4}\right)\right| = \cos\left(\frac{\pi}{4}\cos\psi - \frac{\pi}{4}\right)$$

This is shown sketched in Figure 9.10(c). It has a maximum along $\psi = 0$ and null along $\psi = \pi$. Again, this is an endfire pattern, but directed to one side. This case is the same as the one considered in Section 4.5.

Case 4: $d = \lambda, \alpha = 0$. The group pattern is

$$\left|\cos\left(\frac{\beta\lambda}{2}\cos\psi\right)\right| = |\cos(\pi\cos\psi)|$$

This is shown sketched in Figure 9.10(d). It has maxima along $\psi = 0°, 90°, 180°$, and $270°$ and nulls along $\psi = 60°, 120°, 240°$, and $300°$.

Proceeding further, we can obtain the resultant pattern for an array of two Hertzian dipoles by multiplying the unit pattern by the group pattern. Thus, recalling that the unit pattern for the Hertzian dipole is $\sin\theta$ in the plane of the dipole and considering values of $\lambda/2$ and 0 for d and α, respectively, for which the group pattern is given in Figure 9.10(a), we obtain the resultant pattern in the xz-plane, as shown in Figure 9.11(a). In the xy-plane, that is, the plane normal to the axis of the dipole, the unit pattern is a circle and hence the resultant pattern is the same as the group pattern, as illustrated in Figure 9.11(b).

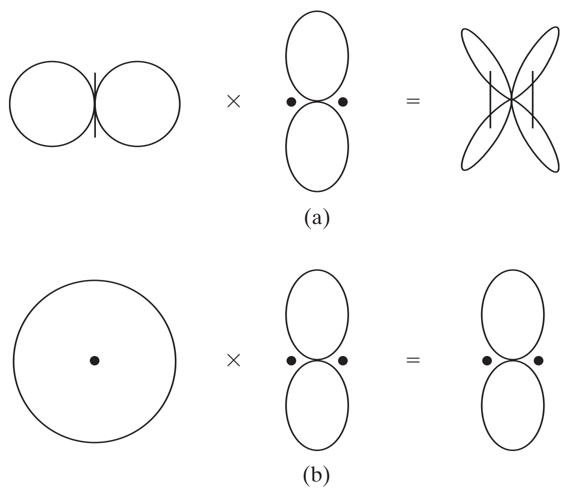


FIGURE 9.11

Determination of the resultant pattern of an antenna array by multiplication of unit and group patterns.

Example 9.3

The procedure of multiplication of the unit and group patterns to obtain the resultant pattern illustrated in Example 9.2 can be extended to an array containing any number of antennas. Let us, for example, consider a linear array of four isotropic antennas spaced $\lambda/2$ apart and fed in phase, as shown in Figure 9.12(a), and obtain the resultant pattern.

To obtain the resultant pattern of the four-element array, we replace it by a two-element array of spacing λ, as shown in Figure 9.12(b), in which each element forms a unit representing a two-element array of spacing $\lambda/2$. The unit pattern is then the pattern shown in Figure 9.10(a). The group pattern, which is the pattern of two isotropic radiators having $d = \lambda$ and $\alpha = 0$, is the pattern given in Figure 9.10(d). The resultant pattern of the four-element array is the product of these two patterns, as illustrated in Figure 9.12(c). If the individual elements of the four-element array are not isotropic, then this pattern becomes the group pattern for the determination of the new resultant pattern.

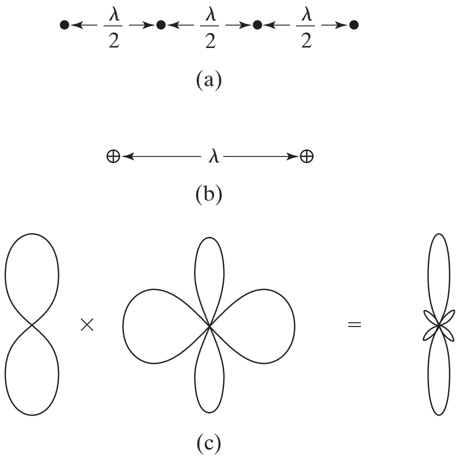


FIGURE 9.12

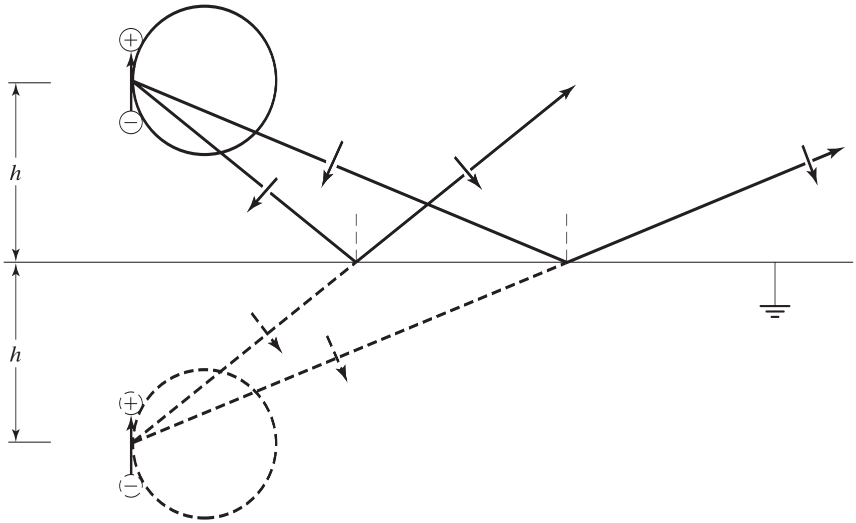
Determination of the resultant pattern for a linear array of four isotropic antennas.

9.5 IMAGE ANTENNAS

Thus far, we have considered the antennas to be situated in an unbounded medium so that the waves radiate in all directions from the antenna without giving rise to reflections from any obstacles. In practice, however, we have to consider the effect of the ground even if no other obstacles are present. To do this, it is reasonable to assume that the ground is a perfect conductor. Hence, in this section we shall consider antennas situated above an infinite plane, perfect-conductor surface and introduce the concept of image sources, a technique that is also useful in solving static field problems.

Thus, let us consider a Hertzian dipole oriented vertically and located at a height h above a plane, perfect-conductor surface, as shown in Figure 9.13(a). Since no waves can penetrate into the perfect conductor, as we learned in Section 5.5, the waves radiated from the dipole onto the conductor give rise to reflected waves, as shown in Figure 9.13(a) for two directions of incidence. For a given incident wave onto the conductor surface, the angle of reflection is equal to the angle of incidence, as can be seen intuitively from the following reasons: (a) The reflected wave must propagate away from the conductor surface, (b) the apparent wavelengths of the incident and reflected waves parallel to the conductor surface must be equal, and (c) the tangential component of the resultant electric field on the conductor surface must be zero, which also determines the polarity of the reflected wave electric field.

If we now produce the directions of propagation of the two reflected waves backward, they meet at a point which is directly beneath the dipole and at the same distance h below the conductor surface as the dipole is above it. Thus, the reflected waves appear to be originating from an antenna, which is the *image* of the actual antenna about the conductor surface. This image antenna must also be a vertical antenna since in order for the boundary condition of zero tangential electric field to be satisfied at all points on the conductor surface, the image antenna must have the same radiation pattern as that of the actual antenna, as shown in Figure 9.13(a). In particular, the current in the image antenna must be directed in the same sense as that in the actual antenna to be consistent with the polarity of the reflected wave electric field. It can therefore be



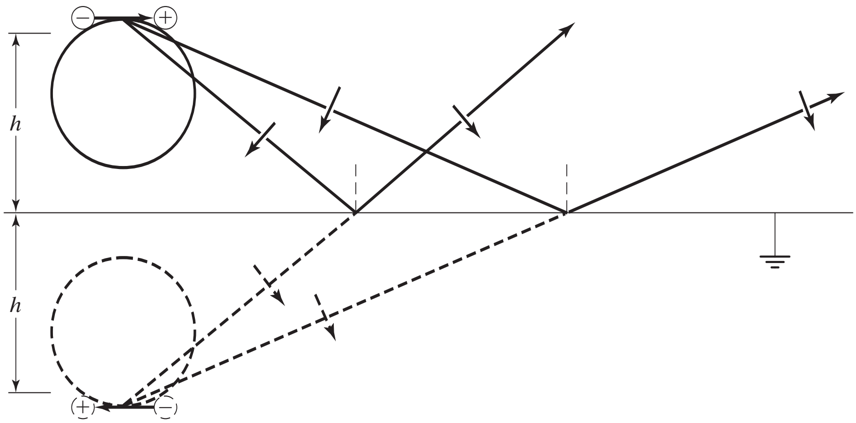


FIGURE 9.13

For illustrating the concept of image antennas. (a) Vertical Hertzian dipole and (b) horizontal Hertzian dipole above a plane, perfect-conductor surface.

seen that the charges associated with the image dipole have signs opposite to those of the corresponding charges associated with the actual dipole.

A similar reasoning can be applied to the case of a horizontal dipole above a perfect conductor surface, as shown in Figure 9.13(b). Here it can be seen that the current in the image antenna is directed in the opposite sense to that in the actual antenna. This again results in charges associated with the image dipole having signs opposite to those of the corresponding charges associated with the actual dipole. In fact, this is always the case.

From the foregoing discussion it can be seen that the field due to an antenna in the presence of the conductor is the same as the resultant field of the array formed by the actual antenna and the image antenna. There is, of course, no field inside the conductor. The image antenna is only a virtual antenna that seves to simplify the field determination outside the conductor. The simplicity arises from the fact that we can use the knowledge gained on antenna arrays in the previous section to determine the radiation pattern. Thus, for example, for a vertical Hertzian dipole at a height of $\lambda/2$ above the conductor surface, the radiation pattern in the vertical plane is the product of the unit pattern, which is the radiation pattern of the single dipole in the plane of its axis, and the group pattern corresponding to an array of two isotropic radiators spaced λ apart and fed in phase. This multiplication and the resultant pattern are illustrated in Figure 9.14. The radiation patterns for the case of the horizontal dipole can be obtained in a similar manner.

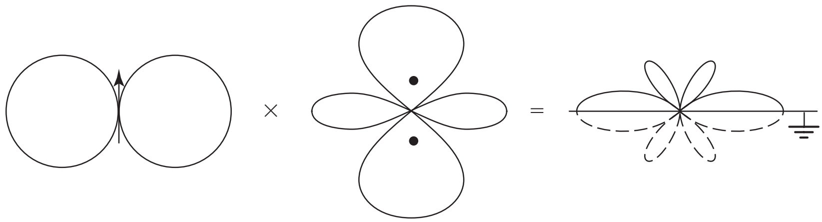


FIGURE 9.14

Determination of radiation pattern in the vertical plane for a vertical Hertzian dipole above a plane, perfect-conductor surface.

9.6 RECEIVING PROPERTIES

Thus far, we have considered the radiating, or transmitting, properties of antennas. Fortunately, it is not necessary to repeat all the derivations for the discussion of the receiving properties of antennas, since reciprocity dictates that the receiving pattern of an antenna be the same as its transmitting pattern. To illustrate this in simple terms without going through the general proof of reciprocity, let us consider a Hertzian dipole situated at the origin and directed along the z-axis, as shown in Figure 9.15. We know that the radiation pattern is then given by $\sin\theta$ and that the polarization of the radiated field is such that the electric field is in the plane of the dipole axis.

To investigate the receiving properties of the Hertzian dipole, we assume that it is situated in the radiation field of a second antenna so that the incoming waves are essentially uniform plane waves. Thus, let us consider a uniform plane wave with its electric field $\mathbf{E}$ in the plane of the dipole and incident on the dipole at an angle θ with its axis, as shown in Figure 9.15. Then the component of the incident electric field parallel to the dipole is $E\sin\theta$. Since the dipole is infinitesimal in length, the voltage induced in the dipole, which is the line integral of the electric field intensity along the length of the dipole, is simply equal to $(E\sin\theta)\,dl$ or to $E\,dl\sin\theta$. This indicates that for a given amplitude of the incident wave field, the induced voltage in the dipole is proportional to $\sin\theta$. Furthermore, for an incident uniform plane wave having its electric field normal to

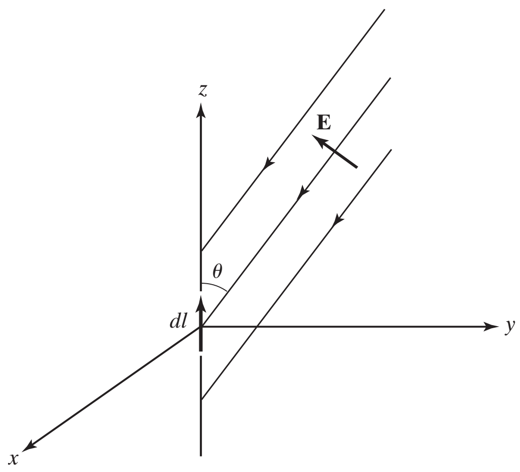


FIGURE 9.15

For investigating the receiving properties of a Hertzian dipole.

the dipole axis, the voltage induced in the dipole is zero, that is, the dipole does not respond to polarization with electric field normal to the plane of its axis. These properties are reciprocal to the transmitting properties of the dipole. Since an arbitrary antenna can be decomposed into a series of Hertzian dipoles, it then follows that reciprocity holds for an arbitrary antenna. Thus, any transmitting antenna can be used as a receiving antenna, and vice versa.

We shall now briefly consider the loop antenna, a common type of receiving antenna. A simple form of loop antenna consists of a circular loop of wire with a pair of terminals. We shall orient the circular loop antenna with its axis aligned with the z-axis, as shown in Figure 9.16, and we shall assume that it is electrically short, that is, its dimensions are small compared to the wavelength of the incident wave, so that the spatial variation of the field over the area of the loop is negligible. For a uniform plane wave incident on the loop, we can find the voltage induced in the loop, that is, the line integral of the electric field intensity around the loop, by using Faraday's law. Thus, if **H**

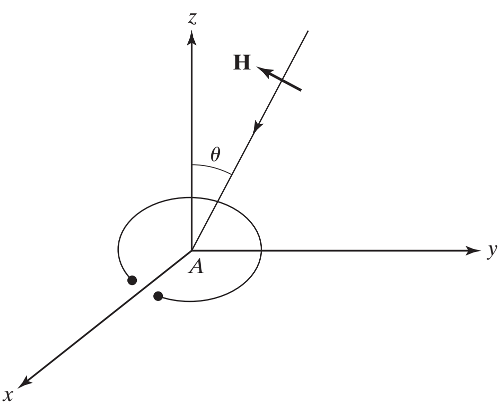


FIGURE 9.16

A circular loop antenna.

is the magnetic field intensity associated with the wave, the magnitude of the induced voltage is given by

$$|V| = \left| -\frac{d}{dt} \int_{\substack{\text{area of} \\ \text{the loop}}} \mathbf{B} \cdot d\mathbf{S} \right|$$

$$= \left| -\mu \frac{d}{dt} \int_{\substack{\text{area of} \\ \text{the loop}}} \mathbf{H} \cdot dS\, \mathbf{a}_z \right|$$

$$= \mu A \left| \frac{\partial H_z}{\partial t} \right| \tag{9.52}$$

where A is the area of the loop. Hence the loop does not respond to a wave having its magnetic field entirely parallel to the plane of the loop, that is, normal to the axis of the loop.

For a wave having its magnetic field in the plane of the axis of the loop, and incident on the loop at an angle θ with its axis, as shown in Figure 9.16, $H_z = H \sin \theta$, and hence the induced voltage has a magnitude

$$|V| = \mu A \left| \frac{\partial H}{\partial t} \right| \sin \theta \tag{9.53}$$

Thus, the receiving pattern of the loop antenna is given by $\sin \theta$, same as that of a Hertzian dipole aligned with the axis of the loop antenna. The loop antenna, however, responds best to polarization with magnetic field in the plane of its axis, whereas the Hertzian dipole responds best to polarization with electric field in the plane of its axis.

Example 9.4

The directional properties of a receiving antenna can be used to locate the source of an incident signal. To illustrate the principle, let us consider two vertical loop antennas, numbered 1 and 2, situated on the x-axis at $x = 0$ m and $x = 200$ m, respectively. By rotating the loop antennas about the vertical (z-axis), it is found that no (or minimum) signal is induced in antenna 1 when it is in the xz-plane and in antenna 2 when it is in a plane making an angle of 5° with the axis, as shown by the top view in Figure 9.17. Let us find the location of the source of the signal.

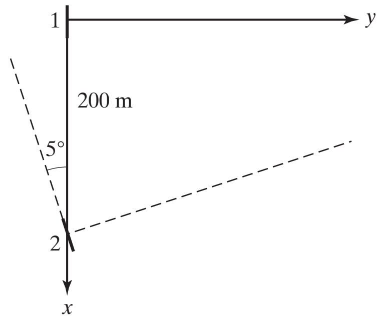


FIGURE 9.17

Top view of two loop antennas used to locate the source of an incident signal.

Since the receiving properties of a loop antenna are such that no signal is induced for a wave arriving along its axis, the source of the signal is located at the intersection of the axes of the two loops when they are oriented so as to receive no (or minimum) signal. From simple geometrical considerations, the source of the signal is therefore located on the y-axis at $y = 200/\tan 5°$ or 2.286 km.

A useful parameter associated with the receiving properties of an antenna is the effective area, denoted A_e and defined as the ratio of the time-average power delivered to a matched load connected to the antenna to the time-average power density of the appropriately polarized incident wave at the antenna. The matched condition is achieved when the load impedance is equal to the complex conjugate of the antenna impedance.

Let us consider the Hertzian dipole and derive the expression for its effective area. First, with reference to the equivalent circuit shown in Figure 9.18, where $\bar{V}_{oc}$ is the open-circuit voltage induced between the terminals of the antenna, $\bar{Z}_A = R_A + jX_A$ is the antenna impedance, and $\bar{Z}_L = \bar{Z}_A^*$ is the load impedance, we note that the time-average power delivered to the matched load is

$$P_R = \frac{1}{2}\left(\frac{|\bar{V}_{oc}|}{2R_A}\right)^2 R_A = \frac{|\bar{V}_{oc}|^2}{8R_A} \tag{9.54}$$

For a Hertzian dipole of length l, the open-circuit voltage is

$$\bar{V}_{oc} = \bar{E}l \tag{9.55}$$

where $\bar{E}$ is the electric field of an incident wave linearly polarized parallel to the dipole axis. Substituting (9.55) into (9.54), we get

$$P_R = \frac{|\bar{E}|^2 l^2}{8R_A} \tag{9.56}$$

For a lossless dipole, $R_A = R_{rad} = 80\pi^2(l/\lambda)^2$, so that

$$P_R = \frac{|\bar{E}|^2\lambda^2}{640\pi^2} \tag{9.57}$$

The time-average power density at the antenna is

$$\frac{|\bar{E}|^2}{2\eta_0} = \frac{|\bar{E}|^2}{240\pi} \tag{9.58}$$

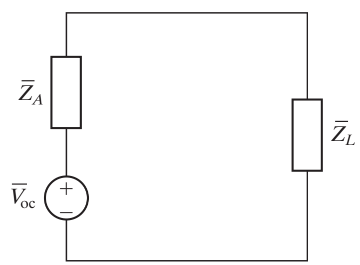


FIGURE 9.18

Equivalent circuit for a receiving antenna connected to a load.

Thus, the effective area is

$$A_e = \frac{|\bar{E}|^2\lambda^2/640\pi^2}{|\bar{E}|^2/240\pi} = \frac{3\lambda^2}{8\pi} \tag{9.59}$$

or

$$A_e = 0.1194\lambda^2 \tag{9.60}$$

In practice, R_A is greater than R_{rad} due to losses in the antenna, and the effective area is less than that given by (9.60). Rewriting (9.59) as

$$A_e = 1.5 \times \frac{\lambda^2}{4\pi}$$

and recalling that the directivity of the Hertzian dipole is 1.5, we observe that

$$A_e = \frac{\lambda^2}{4\pi}D \tag{9.61}$$

Although we have obtained this result for a Hertzian dipole, it can be shown that it holds for any antenna.

We shall now derive the *Friis transmission formula*, an important equation in making communication link calculations. To do this, let us consider two antennas, one transmitting and the other receiving, separated by a distance d. Let us assume that the antennas are oriented and polarization matched so as to maximize the received signal. Then if P_T is the transmitter power radiated by the transmitting antenna, the power density at the receiving antenna is $(P_T/4\pi d^2)D_T$, where D_T is the directivity of the transmitting antenna. The power received by a matched load connected to the terminals of the receiving antenna is then given by

$$P_R = \frac{P_T D_T}{4\pi d^2}A_{eR} \tag{9.62}$$

where A_{eR} is the effective area of the receiving antenna. Thus, the ratio of P_R to P_T is given by

$$\frac{P_R}{P_T} = \frac{D_T A_{eR}}{4\pi d^2} \tag{9.63}$$

Denoting A_{eT} to be the effective area of the transmitting antenna if it were receiving, and using (9.61), we obtain

$$\frac{P_R}{P_T} = \frac{A_{eT}A_{eR}}{\lambda^2 d^2} \tag{9.64}$$

Equation (9.64) is the Friis transmission formula. It gives the maximum value of P_R/P_T for a given d and for a given pair of transmitting and receiving antennas. If the antennas are not oriented to receive the maximum signal, or if a polarization mismatch exists, or if the receiving antenna is not matched to its load, P_R/P_T would be less than that given by (9.64). Losses in the antennas would also decrease the value of P_R/P_T.

An alternative formula to (9.64) is obtained by substituting for A_{eR} in (9.63) in terms of the directivity D_R of the receiving antenna if it were used for transmitting. Thus, we obtain

$$\frac{P_R}{P_T} = \frac{D_T D_R \lambda^2}{16\pi^2 d^2} \tag{9.65}$$

SUMMARY

In this chapter we studied the principles of antennas. We first introduced the Hertzian dipole, which is an elemental wire antenna, and derived the complete electromagnetic field due to the Hertzian dipole by employing an intuitive approach based on the knowledge gained in the previous chapters. For a Hertzian dipole of length dl, oriented along the z-axis at the origin, and carrying current

$$I(t) = I_0 \cos \omega t$$

we found the complete electromagnetic field to be given by

$$\mathbf{E} = \frac{2I_0\,dl \cos\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2}\right]\mathbf{a}_r + \frac{I_0\,dl \sin\theta}{4\pi\epsilon\omega}\left[\frac{\sin(\omega t - \beta r)}{r^3} + \frac{\beta\cos(\omega t - \beta r)}{r^2} - \frac{\beta^2\sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\theta$$

$$\mathbf{H} = \frac{I_0\,dl \sin\theta}{4\pi}\left[\frac{\cos(\omega t - \beta r)}{r^2} - \frac{\beta\sin(\omega t - \beta r)}{r}\right]\mathbf{a}_\phi$$

where $\beta = \omega\sqrt{\mu\epsilon}$ is the phase constant.

For $\beta r \gg 1$ or for $r \gg \lambda/2\pi$, the only important terms in the complete field expressions are the $1/r$ terms, since the remaining terms are negligible compared to these terms. Thus for $r \gg \lambda/2\pi$, the Hertzian dipole fields are given by

$$\mathbf{E} = -\frac{\eta\beta I_0\,dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\theta$$

$$\mathbf{H} = -\frac{\beta I_0\,dl \sin\theta}{4\pi r}\sin(\omega t - \beta r)\,\mathbf{a}_\phi$$

where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance of the medium. These fields, known as the radiation fields, correspond to locally uniform plane waves radiating away from the dipole and, in fact, are the only components of the complete fields contributing to the time-average radiated power. We found the time-average power radiated by the Hertzian dipole to be given by

$$\langle P_{\text{rad}} \rangle = \frac{1}{2} I_0^2 \left[\frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2\right]$$

and identified the quantity inside the brackets to be its radiation resistance. The radiation resistance, R_{rad}, of an antenna is the value of a fictitious resistor that will dissipate the same amount of time-average power as that radiated by the antenna when a current of the same peak amplitude as that in the antenna is passed through it. Thus, for the Hertzian dipole,

$$R_{\text{rad}} = \frac{2\pi\eta}{3}\left(\frac{dl}{\lambda}\right)^2$$

We then examined the directional characteristics of the radiation fields of the Hertzian dipole, as indicated by the factor $\sin\theta$ in the field expressions and hence by the factor $\sin^2\theta$ for the power density. We discussed the radiation patterns and introduced the concept of the directivity of an antenna. The directivity, D, of an antenna is defined as the ratio of the maximum power density radiated by the antenna to the average power density. For the Hertzian dipole,

$$D = 1.5$$

For the general case of a power density pattern $f(\theta, \phi)$, the directivity is given by

$$D = 4\pi \frac{[f(\theta, \phi)]_{\max}}{\displaystyle\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} f(\theta, \phi)\sin\theta\, d\theta\, d\phi}$$

As an illustration of obtaining the radiation fields due to a wire antenna of arbitrary length and arbitrary current distribution by representing it as a series of Hertzian dipoles and using superposition, we considered the example of a half-wave dipole and derived its radiation fields. We found that for a center-fed half-wave dipole of length $L(=\lambda/2)$, oriented along the z-axis with its center at the origin, and having the current distribution given by

$$I(z) = I_0 \cos\frac{\pi z}{L}\cos\omega t \qquad \text{for } -\frac{L}{2} < z < \frac{L}{2}$$

the radiation fields are

$$\mathbf{E} = -\frac{\eta I_0}{2\pi r}\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\sin\left(\omega t - \frac{\pi}{L}r\right)\mathbf{a}_\theta$$

$$\mathbf{H} = -\frac{I_0}{2\pi r}\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\sin\left(\omega t - \frac{\pi}{L}r\right)\mathbf{a}_\phi$$

From these, we sketched the radiation patterns and computed the radiation resistance and the directivity of the half-wave dipole to be

$$R_{\text{rad}} = 73 \text{ ohms} \quad \text{for free space}$$
$$D = 1.642$$

We discussed antenna arrays and introduced the technique of obtaining the resultant radiation pattern of an array by multiplication of the unit and the group patterns. For an array of two antennas having the spacing d and fed with currents of equal amplitude but differing in phase by α, we found the group pattern for the fields to be $|\cos[(\beta d \cos\psi + \alpha)/2]|$, where ψ is the angle measured from the axis of the array, and we investigated the group patterns for several pairs of values of d and α. For example, for $d = \lambda/2$ and $\alpha = 0$, the pattern corresponds to maximum radiation broadside to the axis of the array, whereas for $d = \lambda/2$ and $\alpha = \pi$, the pattern corresponds to maximum radiation endfire to the axis of the array.

To take into account the effect of ground on antennas, we introduced the concept of an image antenna in a perfect conductor and discussed the application of the array techniques in conjunction with the actual and the image antennas to obtain the radiation pattern of the actual antenna in the presence of the ground.

Finally, we discussed receiving properties of antennas. In particular, (1) we discussed the reciprocity between the receiving and radiating properties of an antenna by considering the simple case of a Hertzian dipole, (2) we considered the loop antenna and illustrated the application of its directional properties for locating the source of a radio signal, and (3) we introduced the effective area concept and derived the Friis transmission formula.

REVIEW QUESTIONS

9.1. What is a Hertzian dipole?

9.2. Discuss the time-variations of the current and charges associated with the Hertzian dipole.

9.3. Briefly describe the spherical coordinate system.

9.4. Explain why it is simpler to use the spherical coordinate system to find the fields due to the Hertzian dipole.

9.5. Discuss the reasoning associated with the intuitive extension of the fields due to the time-varying current and charges of the Hertzian dipole based on time-varying electromagnetic phenomena.

9.6. Explain the reason for the inconsistency with Maxwell's equations of the intuitively derived fields due to the time-varying current and charges of the Hertzian dipole.

9.7. Briefly outline the reasoning used for the removal of the inconsistency with Maxwell's equations of the intuitively derived fields due to the Hertzian dipole.

9.8. Discuss the characteristics of the complete electromagnetic field due to the Hertzian dipole.

9.9. Consult an appropriate reference book and compare the procedure used for obtaining the electromagnetic field due to the Hertzian dipole with the procedure used here.

9.10. What are radiation fields? Why are they important?

9.11. Discuss the characteristics of the radiation fields.

9.12. Define the radiation resistance of an antenna.

9.13. Why is the expression for the radiation resistance of a Hertzian dipole not valid for a linear antenna of any length?

9.14. Explain why power lines are not effective radiators.

9.15. What is a radiation pattern?

9.16. Discuss the radiation pattern for the power density due to the Hertzian dipole.

9.17. Define the directivity of an antenna. What is the directivity of a Hertzian dipole?

9.18. What is the directivity of a fictitious antenna that radiates equally in all directions into one hemisphere?

9.19. How do you find the radiation fields due to an antenna of arbitrary length and arbitrary current distribution?

9.20. Discuss the evolution of the half-wave dipole from an open-circuited transmission line.

9.21. Justify the approximations involved in evaluating the integrals in the determination of the radiation fields due to the half-wave dipole.

9.22. What are the values of the radiation resistance and the directivity for a half-wave dipole?

9.23. What is an antenna array?

9.24. Justify the approximations involved in the determination of the resultant field of an array of two antennas.

9.25. Why is it that the distances r_1 and r_2 in the phase factors in equations (8.47a) and (8.47b) cannot be set equal to r, but the same quantities in the amplitude factors can be set equal to r?

9.26. What is an array factor? Provide a physical explanation for the array factor.

9.27. Discuss the concept of unit and group patterns and their multiplication to obtain the resultant pattern of an array.

9.28. Distinguish between broadside and endfire radiation patterns.

9.29. Discuss the concept of an image antenna to find the field of an antenna in the vicinity of a perfect conductor.

9.30. What determines the sense of the current flow in an image antenna relative to that in the actual antenna?

9.31. How does the concept of an image antenna simplify the determination of the radiation pattern of an antenna above a perfect-conductor surface?

9.32. Discuss the reciprocity associated with the transmitting and receiving properties of an antenna. Can you think of a situation in which reciprocity does not hold?

9.33. What is the receiving pattern of a loop antenna?

9.34. How should you orient a loop antenna to receive (a) a maximum signal and (b) a minimum signal?

9.35. Discuss the application of the directional receiving properties of a loop antenna in the location of the source of a radio signal.

9.36. How is the effective area of a receiving antenna defined?

9.37. Outline the derivation of the expression for the effective area of a Hertzian dipole.

9.38. Discuss the derivation of the Friis transmission formula.

PROBLEMS

9.1. The electric dipole moment associated with a Hertzian dipole of length 0.1 m is given by

$$\mathbf{p} = 10^{-9} \sin 2\pi \times 10^7 \, t \, \mathbf{a}_z \text{ C-m}$$

Find the current in the dipole.

9.2. Evaluate the curl of $\mathbf{E}$ given by equation (9.12a) and show that it is not equal to $-\mu \dfrac{\partial \mathbf{H}}{\partial t}$, where $\mathbf{H}$ is given by equation (9.12b).

9.3. Show that in the limit $\omega \to 0$, the complete field expressions given by equations (9.23a) and (9.23b) tend to equations (9.12a) and (9.12b), respectively.

9.4. Show that the radiation fields given by equations (9.25a) and (9.25b) do not by themselves satisfy both of Maxwell's curl equations.

9.5. Find the value of r at which the amplitude of the radiation field term in equation (9.23a) is equal to the resultant amplitude of the remaining two terms in the θ-component.

9.6. Obtain the Poynting vector corresponding to the complete electromagnetic field due to the Hertzian dipole and show that the $1/r^3$ and $1/r^2$ terms do not contribute to the time-average power flow from the dipole.

9.7. A straight wire of length 1 m situated in free space carries a uniform current $10 \cos 4\pi \times 10^6 t$ A. (a) Calculate the amplitude of the electric field intensity at a distance of 10 km in a direction at right angle to the wire. (b) Calculate the radiation resistance and the time-average power radiated by the wire.

9.8. Compute the radiation resistance per kilometer length of a straight power-line wire. Comment on the effectiveness of the power line as a radiator.

9.9. Find the time-average power required to be radiated by a Hertzian dipole in order to produce an electric field intensity of peak amplitude 0.01 V/m at a distance of 1 km broadside to the dipole.

9.10. A Hertzian dipole situated at the origin and oriented along the x-axis carries a current $I_1 = I_0 \cos \omega t$. A second Hertzian dipole, having the same length and also situated at the origin but oriented along the z-axis, carries a current $I_2 = I_0 \sin \omega t$. Find the polarization of the radiated electric field at (a) a point on the x-axis, (b) a point on the z-axis, (c) a point on the y-axis, and (d) a point on the line $x = y, z = 0$.

9.11. Find the ratio of the currents in two antennas having directivities D_1 and D_2 and radiation resistances $R_{\text{rad}\,1}$ and $R_{\text{rad}\,2}$ for which the maximum radiated power densities are equal.

9.12. The radiation pattern for the power density of an antenna located at the origin is dependent on θ in the manner $\sin^4 \theta$. Find the directivity of the antenna.

9.13. The radiation pattern for the power density of an antenna located at the origin is dependent on θ in the manner

$$f(\theta, \phi) = \begin{cases} \csc^2 \theta & \text{for } \pi/6 \le \theta \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

Find the directivity of the antenna.

9.14. In Figure 9.7, let $L = 2$ m, and investigate the variations of r' and $\pi r'/L$ for $-L/2 < z' < L/2$ for (a) a point in the xy-plane at $r = 1$ km and (b) a point on the z-axis at $r = 1$ km.

9.15. By dividing the interval $0 < \theta < \pi/2$ into nine equal parts, numerically compute the value of

$$\int_{\theta=0}^{\pi/2} \frac{\cos^2 [(\pi/2) \cos \theta]}{\sin \theta}\, d\theta$$

9.16. Complete the missing steps in the evaluation of the integral in equation (9.39a).

9.17. Find the time-average power required to be radiated by a half-wave dipole in order to produce an electric field intensity of peak amplitude 0.01 V/m at a distance of 1 km broadside to the dipole.

9.18. Compare the correct value of the radiation resistance of the half-wave dipole with the incorrect value that would result from using the expression for the radiation resistance of the Hertzian dipole.

9.19. A short dipole is a center-fed straight wire antenna having a length that is small compared to a wavelength. The amplitude of the current distribution can then be approximated as decreasing linearly from a maximum at the center to zero at the ends. Thus, for a short dipole of length L lying along the z-axis between $z = -L/2$ and $z = L/2$, the current distribution is given by

$$I(z) = \begin{cases} I_0\left(1 + \dfrac{2z}{L}\right)\cos\omega t & \text{for } -\dfrac{L}{2} < z < 0 \\ I_0\left(1 - \dfrac{2z}{L}\right)\cos\omega t & \text{for } 0 < z < \dfrac{L}{2} \end{cases}$$

(a) Obtain the radiation fields of the short dipole. (b) Find the radiation resistance and the directivity of the short dipole.

9.20. For the array of two antennas of Example 9.2, find and sketch the group patterns for (a) $d = \lambda/4$, $\alpha = \pi/2$ and (b) $d = 2\lambda$, $\alpha = 0$.

9.21. For the array of two antennas of Example 9.2, having $d = \lambda/4$, find the value of α for which the maxima of the group pattern are directed along $\psi = \pm 60°$, and then sketch the group pattern.

9.22. Obtain the resultant pattern for a linear array of eight isotropic antennas, spaced $\lambda/2$ apart, carrying equal currents, and fed in phase.

9.23. Obtain the resultant pattern for a linear array of three isotropic antennas, spaced $\lambda/2$ apart, carrying unequal currents in the ratio 1 : 2 : 1, and fed in phase.

9.24. For the array of two Hertzian dipoles of Figure 9.9, find and sketch the resultant pattern in the xz-plane for $d = \lambda/2$ and $\alpha = \pi$.

9.25. For the array of two Hertzian dipoles of Figure 9.9, find and sketch the resultant pattern in the xz-plane for $d = \lambda/4$ and $\alpha = -\pi/2$.

9.26. For a horizontal Hertzian dipole at a height $\lambda/4$ above a plane, perfect-conductor surface, find and sketch the radiation pattern in (a) the vertical plane perpendicular to the axis of the antenna and (b) the vertical plane containing the axis of the antenna.

9.27. For a vertical antenna of length $\lambda/4$ above a plane, perfect-conductor surface, find (a) the radiation pattern in the vertical plane and (b) the directivity.

9.28. A Hertzian dipole is situated parallel to a corner reflector, which is an arrangement of two plane, perfect conductors at right angles to each other, as shown by the cross-sectional view in Figure 9.19. (a) Locate the image antennas required to satisfy the boundary conditions on the corner reflector surface. (b) Find and sketch the radiation pattern in the cross-sectional plane.

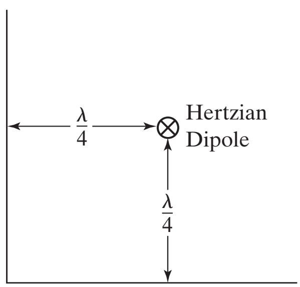


FIGURE 9.19

For Problem 9.28.

9.29. If the Hertzian dipole in Figure 9.19 is situated at a distance $\lambda/2$ from the corner and equidistant from the two planes, find the ratio of the radiation field at a point broadside to the dipole and away from the corner to the radiation field in the absence of the corner reflector.

9.30. An arrangement of two identical Hertzian dipoles situated at the origin and oriented along the x- and y-axes, known as the turnstile antenna, is used for receiving circularly polarized signals arriving along the z-axis. Determine how you would combine the voltages induced in the two dipoles so that the turnstile antenna is responsive to circular polarization rotating in the clockwise sense as viewed by the antenna but not to that of the counterclockwise sense of rotation.

9.31. A vertical loop antenna of area 1 m^2 is situated at a distance of 10 km from a vertical wire antenna of length $\lambda/4$ above a perfectly conducting ground (directivity = 3.28; see Problem 9.27) radiating at 2 MHz. The loop antenna is oriented so as to maximize the signal induced in it. For a time-average radiated power of 10 kW, find the amplitude of the voltage induced in the loop antenna.

9.32. An interferometer consists of an array of two identical antennas with spacing d. Show that for a uniform plane wave incident on the array at an angle ψ to the axis of the array, as shown in Figure 9.20, the phase difference $\Delta\phi$ between the voltage induced in antenna 1 and the voltage induced in antenna 2 is $(2\pi d/\lambda)\cos\psi$, where λ is the wavelength of the incident wave. For $d = 2\lambda$ and for $\Delta\phi = 30°$, find all possible values of ψ. Take into account the fact that the phase measurement is ambiguous by the amount $\pm 2n\pi$, where n is an integer.

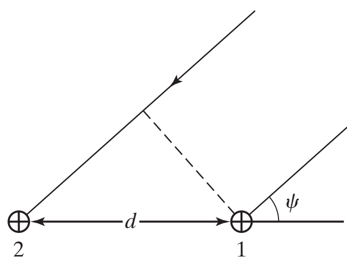


FIGURE 9.20
For Problem 9.32.

9.33. A communication link at a frequency of 30 MHz uses a half-wave dipole for the transmitting antenna and a small loop (directivity equal to 1.5) for the receiving antenna, involving a distance of 100 km. The antennas are oriented so as to receive maximum signal and the receiving antenna is matched to its load. If the received time-average power is to be 1 μW, find the minimum required value of the maximum amplitude I_0 of the current with which the transmitting antenna has to be excited. Assume the antennas to be lossless.