CHAPTER 5

# Wave Propagation in Material Media

In Chapter 4, we introduced wave propagation in free space by considering the infinite plane current sheet of uniform, sinusoidally time-varying current density. We learned that the solution for the electromagnetic field due to the infinite plane current sheet represents uniform plane electromagnetic waves propagating away from the sheet to either side of it. With the knowledge of the principles of uniform plane wave propagation in free space, we are now ready to consider wave propagation in material media, which is our goal in this chapter. Materials contain charged particles that respond to applied electric and magnetic fields and give rise to currents, which modify the properties of wave propagation from those associated with free space.

We shall learn that there are three basic phenomena resulting from the interaction of the charged particles with the electric and magnetic fields. These are conduction, polarization, and magnetization. Although a given material may exhibit all three properties, it is classified as a conductor, a dielectric, or a magnetic material, depending on whether conduction, polarization, or magnetization is the predominant phenomenon. Thus, we shall introduce these three kinds of materials one at a time and develop a set of relations known as the constitutive relations that enable us to avoid the necessity of explicitly taking into account the interaction of the charged particles with the fields. We shall then use these constitutive relations together with Maxwell's equations to first discuss uniform plane wave propagation in a general material medium and then consider several special cases. Finally, we shall derive the *boundary conditions* and use them to study reflection and transmission of uniform plane waves at plane boundaries.

## 5.1 CONDUCTORS AND DIELECTRICS

We recall that the classical model of an atom postulates a tightly bound, positively charged nucleus surrounded by a diffuse cloud of electrons spinning and orbiting around the nucleus. In the absence of an applied electromagnetic field, the force of attraction between the positively charged nucleus and the negatively charged electrons is balanced by the outward centrifugal force to maintain stable electronic orbits. The electrons can be divided into *bound* electrons and *free* or *conduction* electrons.

The bound electrons can be displaced but not removed from the influence of the nucleus. The conduction electrons are constantly under thermal agitation, being released from the parent atom at one point and recaptured by another atom at a different point.

In the absence of an applied field, the motion of the conduction electrons is completely random; the average thermal velocity on a *macroscopic* scale, that is, over volumes large compared with atomic dimensions, is zero so that there is no net current and the electron cloud maintains a fixed position. With the application of an electromagnetic field, an additional velocity is superimposed on the random velocities, predominantly due to the electric force. This causes drift of the average position of the electrons in a direction opposite to that of the applied electric field. Due to the frictional mechanism provided by collisions of the electrons with the atomic lattice, the electrons, instead of accelerating under the influence of the electric field, drift with an average drift velocity proportional in magnitude to the applied electric field. This phenomenon is known as *conduction*, and the resulting current due to the electron drift is known as the *conduction current.*

In certain materials a large number of electrons may take part in the conduction process, but in certain other materials only a very few or negligible number of electrons may participate in conduction. The former class of materials is known as *conductors*, and the latter class is known as *dielectrics* or *insulators.* If the number of free electrons participating in conduction is $N_e$ per cubic meter of the material, then the conduction current density is given by

$$\mathbf{J}_c = N_e e \mathbf{v}_d \tag{5.1}$$

where $e$ is the charge of an electron, and $\mathbf{v}_d$ is the drift velocity of the electrons. The drift velocity varies from one conductor to another, depending on the average time between successive collisions of the electrons with the atomic lattice. It is related to the applied electric field in the manner

$$\mathbf{v}_d = -\mu_e \mathbf{E} \tag{5.2}$$

where $\mu_e$ is known as the *mobility* of the electron. Substituting (5.2) into (5.1), we obtain

$$\mathbf{J}_c = -\mu_e N_e e \mathbf{E} = \mu_e N_e |e| \mathbf{E} \tag{5.3}$$

Semiconductors are characterized by drift of *holes*, that is, vacancies created by detachment of electrons from covalent bonds, in addition to the drift of electrons. If $N_e$ and $N_h$ are the number of electrons and holes, respectively, per cubic meter of the material, and if $\mu_e$ and $\mu_h$ are the electron and hole mobilities, respectively, then the conduction current density in the semiconductor is given by

$$\mathbf{J}_c = (\mu_e N_e |e| + \mu_h N_h |e|) \mathbf{E} \tag{5.4}$$

Defining a quantity $\sigma$, known as the *conductivity* of the material, as given by

$$\sigma = \begin{cases} \mu_e N_e |e| & \text{for conductors} \\ \mu_e N_e |e| + \mu_h N_h |e| & \text{for semiconductors} \end{cases} \tag{5.5}$$

we obtain the simple and important relationship

$$\mathbf{J}_c = \sigma \mathbf{E} \tag{5.6}$$

for the conduction current density in a material. Equation (5.6) is known as Ohm's law applicable at a point from which follows the familiar form of Ohm's law used in circuit theory. The units of $\sigma$ are siemens/meter where a siemen (S) is an ampere per volt. Values of $\sigma$ for a few materials are listed in Table 5.1. In considering electromagnetic wave propagation in conducting media, the conduction current density given by (5.6) must be employed for the current density term on the right side of Ampere's circuital law. Thus, Maxwell's curl equation for **H** for a conducting medium is given by

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} \tag{5.7}$$

TABLE 5.1 Conductivities of Some Materials

| Material | Conductivity S/m | Material | Conductivity S/m |
|---|---|---|---|
| Silver | $6.1 \times 10^7$ | Sea water | 4 |
| Copper | $5.8 \times 10^7$ | Intrinsic germanium | 2.2 |
| Gold | $4.1 \times 10^7$ | Intrinsic silicon | $1.6 \times 10^{-3}$ |
| Aluminum | $3.5 \times 10^7$ | Fresh water | $10^{-3}$ |
| Tungsten | $1.8 \times 10^7$ | Distilled water | $2 \times 10^{-4}$ |
| Brass | $1.5 \times 10^7$ | Dry earth | $10^{-5}$ |
| Solder | $7.0 \times 10^6$ | Bakelite | $10^{-9}$ |
| Lead | $4.8 \times 10^6$ | Glass | $10^{-10} - 10^{-14}$ |
| Constantin | $2.0 \times 10^6$ | Mica | $10^{-11} - 10^{-15}$ |
| Mercury | $1.0 \times 10^6$ | Fused quartz | $0.4 \times 10^{-17}$ |

While conductors are characterized by abundance of *conduction* or *free* electrons that give rise to conduction current under the influence of an applied electric field, in dielectric materials the *bound* electrons are predominant. Under the application of an external electric field, the bound electrons of an atom are displaced such that the centroid of the electron cloud is separated from the centroid of the nucleus. The atom is then said to be *polarized*, thereby creating an *electric dipole*, as shown in Figure 5.1(a). This kind of polarization is called *electronic polarization.* The schematic representation of an electric dipole is shown in Figure 5.1(b). The strength of the dipole is defined by the electric dipole moment **p** given by

$$\mathbf{p} = Q\mathbf{d} \tag{5.8}$$

where **d** is the vector displacement between the centroids of the positive and negative charges, each of magnitude $Q$.

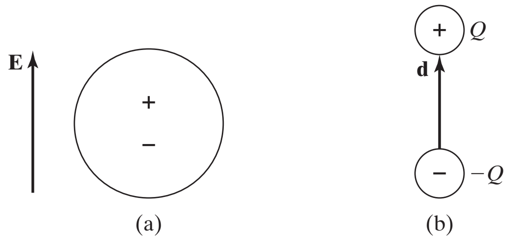


FIGURE 5.1
(a) An electric dipole. (b) Schematic representation of an electric dipole.

In certain dielectric materials, polarization may exist in the molecular structure of the material even under the application of no external electric field. The polarization of individual atoms and molecules, however, is randomly oriented, and hence the net polarization on a *macroscopic* scale is zero. The application of an external field results in torques acting on the *microscopic* dipoles, as shown in Figure 5.2, to convert the initially random polarization into a partially coherent one along the field, on a macroscopic scale. This kind of polarization is known as *orientational polarization.* A third kind of polarization, known as *ionic polarization*, results from the separation of positive and negative ions in molecules formed by the transfer of electrons from one atom to another in the molecule. Certain materials exhibit permanent polarization, that is, polarization even in the absence of an applied electric field. Electrets, when allowed to solidify in the applied electric field, become permanently polarized, and ferroelectric materials exhibit spontaneous, permanent polarization.

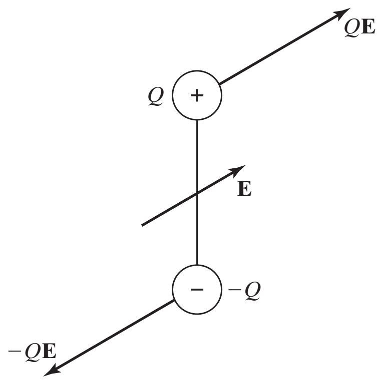


FIGURE 5.2
Torque acting on an electric dipole in an external electric field.

On a macroscopic scale, we define a vector **P**, called the *polarization vector*, as the *electric dipole moment per unit volume*. Thus, if $N$ denotes the number of molecules per unit volume of the material, then there are $N\Delta v$ molecules in a volume $\Delta v$ and

$$\mathbf{P} = \frac{1}{\Delta v}\sum_{j=1}^{N\,\Delta v}\mathbf{p}_j = N\mathbf{p} \tag{5.9}$$

where **p** is the average dipole moment per molecule. The units of **P** are coulomb-meter/meter$^3$ or coulombs per square meter. It is found that for many dielectric materials the polarization vector is related to the electric field **E** in the dielectric in the simple manner given by

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \tag{5.10}$$

where $\chi_e$, a dimensionless parameter, is known as the *electric susceptibility.* The quantity $\chi_e$ is a measure of the ability of the material to become polarized and differs from one dielectric to another.

To discuss the influence of polarization in the dielectric upon electromagnetic wave propagation in the dielectric medium, let us consider the case of the infinite plane current sheet of Figure 4.8, radiating uniform plane waves, except that now the space on either side of the current sheet is a dielectric medium instead of being free space. The electric field in the medium induces polarization. The polarization in turn acts together with other factors to govern the behavior of the electromagnetic field. For the case under consideration, the electric field is entirely in the $x$-direction and uniform in $x$ and $y$. Thus, the induced electric dipoles are all oriented in the $x$-direction, on a macroscopic scale, with the dipole moment per unit volume given by

$$\mathbf{P} = P_x\mathbf{a}_x = \epsilon_0\chi_e E_x\mathbf{a}_x \tag{5.11}$$

where $E_x$ is understood to be a function of $z$ and $t$.

If we now consider an infinitesimal surface of area $\Delta y\ \Delta z$ parallel to the $yz$-plane, we can write $E_x$ associated with that infinitesimal area to be equal to $E_0 \cos \omega t$ where $E_0$ is a constant. The time history of the induced dipoles associated with that area can be sketched for one complete period of the current source, as shown in Figure 5.3. In view of the cosinusoidal variation of the electric field with time, the dipole moment of the individual dipoles varies in a cosinusoidal manner with maximum strength in the positive $x$-direction at $t = 0$, decreasing sinusoidally to zero strength at $t = \pi/2\omega$ and then reversing to the negative $x$-direction, increasing to maximum strength in that direction at $t = \pi/\omega$, and so on.

The arrangement can be considered as two plane sheets of equal and opposite time-varying charges displaced by the amount $\delta$ in the $x$-direction, as shown in Figure 5.4. To find the magnitude of either charge, we note that the dipole moment per unit volume is

$$P_x = \epsilon_0\chi_e E_0 \cos \omega t \tag{5.12}$$

Since the total volume occupied by the dipoles is $\delta\,\Delta y\ \Delta z$, the total dipole moment associated with the dipoles is $\epsilon_0\chi_e E_0 \cos \omega t\ (\delta\,\Delta y\ \Delta z)$. The dipole moment associated with two equal and opposite sheet charges is equal to the magnitude of either sheet charge multiplied by the displacement between the two sheets. Hence, we obtain the magnitude of either sheet charge to be $|\epsilon_0\chi_e E_0 \cos \omega t\ \Delta y\ \Delta z|$. Thus, we have a situation in which a sheet charge $Q_1 = \epsilon_0\chi_e E_0 \cos \omega t\ \Delta y\ \Delta z$ is above the surface and a sheet charge $Q_2 = -Q_1 = -\epsilon_0\chi_e E_0 \cos \omega t\ \Delta y\ \Delta z$ is below the surface. This is equivalent to a current flowing across the surface, since the charges are varying with time.

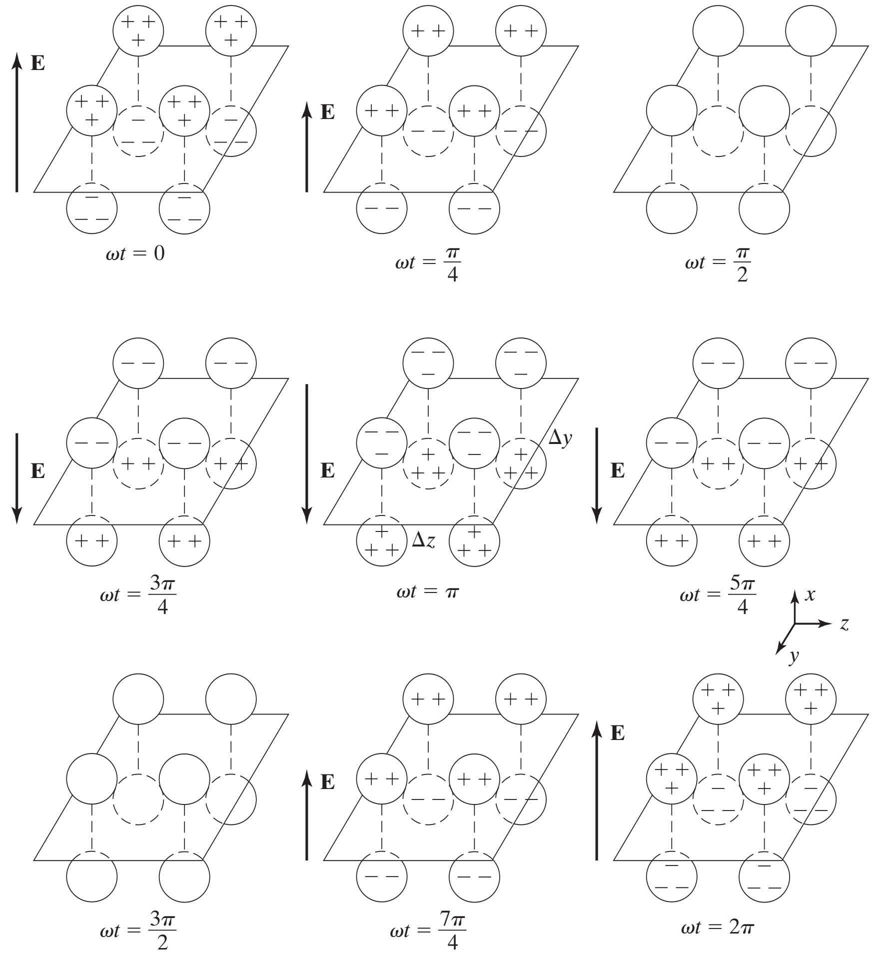


FIGURE 5.3

Time history of induced electric dipoles in a dielectric material under the influence of a sinusoidally time-varying electric field.

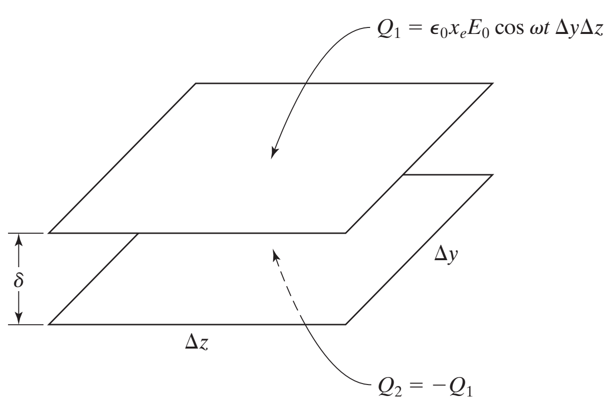


FIGURE 5.4

Two plane sheets of equal and opposite time-varying charges equivalent to the phenomenon depicted in Figure 5.3.

We call this current the *polarization current*, since it results from the time variation of the electric dipole moments induced in the dielectric due to polarization. The polarization current crossing the surface in the positive $x$-direction, that is, from below to above, is

$$I_{px} = \frac{dQ_1}{dt} = -\epsilon_0 \chi_e E_0 \omega \sin \omega t \; \Delta y \; \Delta z \tag{5.13}$$

where the subscript $p$ denotes polarization. By dividing $I_{px}$ by $\Delta y \; \Delta z$ and letting the area tend to zero, we obtain the polarization current density associated with the points on the surface as

$$\begin{aligned} J_{px} &= \lim_{\substack{\Delta y \to 0 \\ \Delta z \to 0}} \frac{I_{px}}{\Delta y \; \Delta z} = -\epsilon_0 \chi_e E_0 \omega \sin \omega t \\ &= \frac{\partial}{\partial t}(\epsilon_0 \chi_e E_0 \cos \omega t) = \frac{\partial P_x}{\partial t} \end{aligned} \tag{5.14}$$

or

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t} \tag{5.15}$$

Although we have deduced this result by considering the special case of the infinite plane current sheet, it is valid in general.

In considering electromagnetic wave propagation in a dielectric medium, the polarization current density given by (5.15) must be included with the current density term on the right side of Ampere's circuital law. Thus, considering Ampere's circuital law in differential form for the general case given by (3.28), we have

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_p + \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E}) \tag{5.16}$$

Substituting (5.15) into (5.16), we get

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{P}}{\partial t} + \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E}) \\ &= \mathbf{J} + \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E} + \mathbf{P}) \end{aligned} \tag{5.17}$$

In order to make (5.17) consistent with the corresponding equation for free space given by (3.28), we now revise the definition of the displacement vector $\mathbf{D}$ to read as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{5.18}$$

Substituting for $\mathbf{P}$ by using (5.10), we obtain

$$\begin{aligned} \mathbf{D} &= \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E} \\ &= \epsilon_0 (1 + \chi_e) \mathbf{E} \\ &= \epsilon_0 \epsilon_r \mathbf{E} \\ &= \epsilon \mathbf{E} \end{aligned} \tag{5.19}$$

where we define

$$\epsilon_r = 1 + \chi_e \tag{5.20}$$

and

$$\epsilon = \epsilon_0 \epsilon_r \tag{5.21}$$

The quantity $\epsilon_r$ is known as the *relative permittivity* or *dielectric constant* of the dielectric, and $\epsilon$ is the *permittivity* of the dielectric. The new definition for **D** permits the use of the same Maxwell's equations as for free space with $\epsilon_0$ replaced by $\epsilon$ and without the need for explicitly considering the polarization current density. The permittivity $\epsilon$ takes into account the effects of polarization, and there is no need to consider them when we use $\epsilon$ for $\epsilon_0$! The relative permittivity is an experimentally measurable parameter and its values for several dielectric materials are listed in Table 5.2.

TABLE 5.2 Relative Permittivities of Some Materials

| Material | Relative Permittivity | Material | Relative Permittivity |
|---|---|---|---|
| Air | 1.0006 | Dry earth | 5 |
| Paper | 2.0–3.0 | Mica | 6 |
| Teflon | 2.1 | Neoprene | 6.7 |
| Polystyrene | 2.56 | Wet earth | 10 |
| Plexiglass | 2.6–3.5 | Ethyl alcohol | 24.3 |
| Nylon | 3.5 | Glycerol | 42.5 |
| Fused quartz | 3.8 | Distilled water | 81 |
| Bakelite | 4.9 | Titanium dioxide | 100 |

Equation (5.19) governs the relationship between **D** and **E** for dielectric materials. Dielectrics for which $\epsilon$ is independent of the magnitude as well as the direction of **E** as indicated by (5.19) are known as *linear isotropic dielectrics.* For certain dielectric materials, each component of the polarization vector can be dependent on all components of the electric field intensity. For such materials, known as *anisotropic dielectric materials*, **D** is not in general parallel to **E**, and the relationship between these two quantities is expressed in the form of a matrix equation, as given by

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{5.22}$$

The square matrix in (5.22) is known as the *permittivity tensor* of the anisotropic dielectric.

## Example 5.1

An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \begin{bmatrix} 7\epsilon_0 & 2\epsilon_0 & 0 \\ 2\epsilon_0 & 4\epsilon_0 & 0 \\ 0 & 0 & 3\epsilon_0 \end{bmatrix}$$

Let us find **D** for several cases of **E**.

Substituting the given permittivity matrix in (5.22), we obtain

$$D_x = 7\epsilon_0 E_x + 2\epsilon_0 E_y$$
$$D_y = 2\epsilon_0 E_x + 4\epsilon_0 E_y$$
$$D_z = 3\epsilon_0 E_z$$

For $\mathbf{E} = E_0 \cos \omega t\, \mathbf{a}_z$, $\mathbf{D} = 3\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_z$; **D** is parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t\, \mathbf{a}_x$, $\mathbf{D} = 7\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_x + 2\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t\, \mathbf{a}_y$, $\mathbf{D} = 2\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_x + 4\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t\, (\mathbf{a}_x + 2\mathbf{a}_y)$, $\mathbf{D} = 11\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_x + 10\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t\, (2\mathbf{a}_x + \mathbf{a}_y)$, $\mathbf{D} = 16\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_x + 8\epsilon_0 E_0 \cos \omega t\, \mathbf{a}_y = 8\epsilon_0 \mathbf{E}$; **D** is parallel to **E** and the dielectric behaves *effectively* in the same manner as an isotropic dielectric having the permittivity $8\epsilon_0$; that is, the *effective permittivity* of the anisotropic dielectric for this case is $8\epsilon_0$.

Thus, we find that in general **D** is not parallel to **E** but for certain polarizations of **E**, **D** is parallel to **E**. These polarizations are known as the characteristic polarizations.

## 5.2 MAGNETIC MATERIALS

The important characteristic of magnetic materials is *magnetization.* Magnetization is the phenomenon by means of which the orbital and spin motions of electrons are influenced by an external magnetic field. An electronic orbit is equivalent to a current loop, which is the magnetic analog of an electric dipole. The schematic representation of a magnetic dipole as seen from along its axis and from a point in its plane are shown in Figures 5.5(a) and 5.5(b), respectively. The strength of the dipole is defined by the magnetic dipole moment **m** given by

$$\mathbf{m} = IA\mathbf{a}_n \tag{5.23}$$

where $A$ is the area enclosed by the current loop and $\mathbf{a}_n$ is the unit vector normal to the plane of the loop and directed in the right-hand sense.

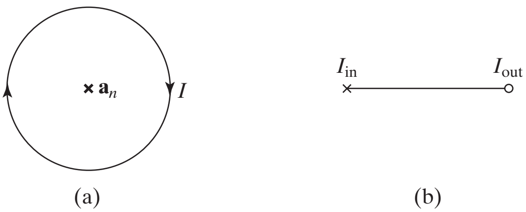


FIGURE 5.5

Schematic representation of a magnetic dipole as seen from (a) along its axis and (b) a point in its plane.

In many materials, the net magnetic moment of each atom is zero, that is, on the average, the magnetic dipole moments corresponding to the various electronic orbital and spin motions add up to zero. An external magnetic field has the effect of inducing a net dipole moment by changing the angular velocities of the electronic orbits, thereby magnetizing the material. This kind of magnetization, known as *diamagnetism*, is in fact prevalent in all materials. In certain materials known as *paramagnetic materials*, the individual atoms possess net nonzero magnetic moments even in the absence of an external magnetic field. These *permanent* magnetic moments of the individual atoms are, however, randomly oriented so that the net magnetization on a macroscopic scale is zero. An applied magnetic field has the effect of exerting torques on the individual permanent dipoles, as shown in Figure 5.6, to convert, on a macroscopic scale, the initially random alignment into a partially coherent one along the magnetic field, that is, with the normal to the current loop directed along the magnetic field. This kind of magnetization is known as *paramagnetism.* Certain materials known as *ferromagnetic*, *antiferromagnetic*, and *ferrimagnetic* materials exhibit permanent magnetization, that is, magnetization even in the absence of an applied magnetic field.

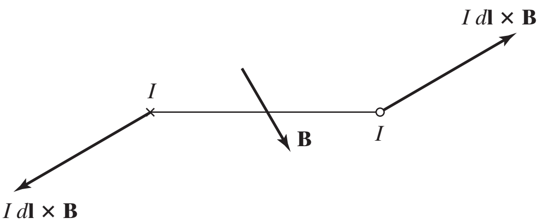


FIGURE 5.6

Torque acting on a magnetic dipole in an external magnetic field.

On a macroscopic scale, we define a vector **M**, called the *magnetization vector*, as the *magnetic dipole moment per unit volume.* Thus, if $N$ denotes the number of molecules per unit volume of the material, then there are $N\Delta v$ molecules in a volume $\Delta v$ and

$$\mathbf{M} = \frac{1}{\Delta v}\sum_{j=1}^{N\Delta v}\mathbf{m}_j = N\mathbf{m} \tag{5.24}$$

where **m** is the average dipole moment per molecule. The units of **M** are ampere-meter$^2$/meter$^3$ or amperes per meter. It is found that for many magnetic materials, the magnetization vector is related to the magnetic field **B** in the material in the simple manner given by

$$\mathbf{M} = \frac{\chi_m}{1 + \chi_m}\frac{\mathbf{B}}{\mu_0} \tag{5.25}$$

where $\chi_m$, a dimensionless parameter, is known as the *magnetic susceptibility.* The quantity $\chi_m$ is a measure of the ability of the material to become magnetized and differs from one magnetic material to another.

To discuss the influence of magnetization in the material on electromagnetic wave propagation in the magnetic material medium, let us consider the case of the infinite plane current sheet of Figure 4.8, radiating uniform plane waves, except that now the space on either side of the current sheet possesses magnetic material properties in addition to dielectric properties. The magnetic field in the medium induces magnetization. The magnetization in turn acts together with other factors to govern the behavior of the electromagnetic field. For the case under consideration, the magnetic field is entirely in the $y$-direction and uniform in $x$ and $y$. Thus, the induced dipoles are all oriented with their axes in the $y$-direction, on a macroscopic scale, with the dipole moment per unit volume given by

$$\mathbf{M} = M_y\mathbf{a}_y = \frac{\chi_m}{1 + \chi_m}\frac{B_y}{\mu_0}\mathbf{a}_y \tag{5.26}$$

where $B_y$ is understood to be a function of $z$ and $t$.

Let us now consider an infinitesimal surface of area $\Delta y\, \Delta z$ parallel to the $yz$-plane and the magnetic dipoles associated with the two areas $\Delta y\, \Delta z$ to the left and to the right of the center of this area, as shown in Figure 5.7(a). Since $B_y$ is a function of $z$, we can assume the dipoles in the left area to have a different moment than the dipoles in the right area for any given time. If the dimension of an individual dipole is $\delta$ in the $x$-direction, then the total dipole moment associated with the dipoles in the left area is $[M_y]_{z-\Delta z/2}\, \delta\, \Delta y\, \Delta z$ and the total dipole moment associated with the dipoles in the right area is $[M_y]_{z+\Delta z/2}\, \delta\, \Delta y\, \Delta z$.

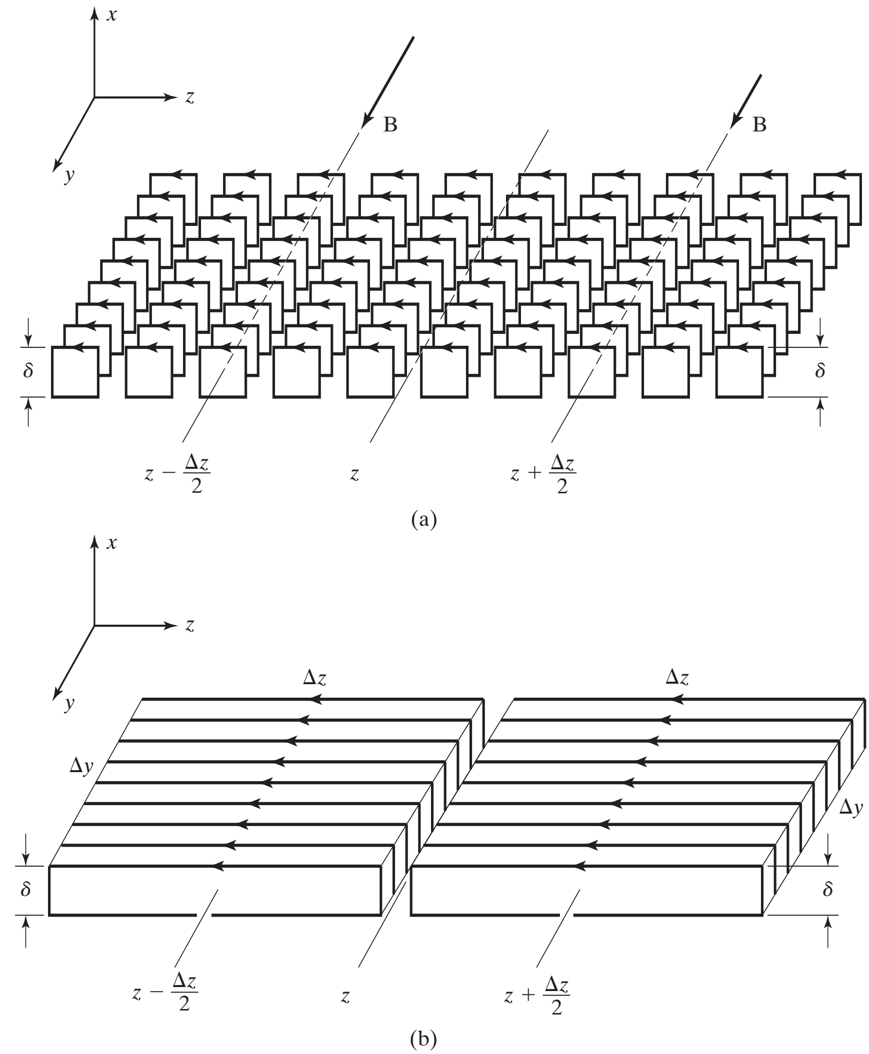


FIGURE 5.7

(a) Induced magnetic dipoles in a magnetic material. (b) Equivalent surface current loops.

The arrangement of dipoles can be considered to be equivalent to two rectangular surface current loops, as shown in Figure 5.7(b), with the left side current loop having a dipole moment $[M_y]_{z-\Delta z/2}\,\delta\,\Delta y\,\Delta z$ and the right side current loop having a dipole moment $[M_y]_{z+\Delta z/2}\,\delta\,\Delta y\,\Delta z$. Since the magnetic dipole moment of a rectangular surface current loop is simply equal to the product of the surface current and the cross-sectional area of the loop, the surface current associated with the left loop is $[M_y]_{z-\Delta z/2}\,\Delta y$ and the surface current associated with the right loop is $[M_y]_{z+\Delta z/2}\,\Delta y$. Thus, we have a situation in which a current equal to $[M_y]_{z-\Delta z/2}\,\Delta y$ is crossing the area $\Delta y\,\Delta z$ in the positive $x$-direction, and a current equal to $[M_y]_{z+\Delta z/2}\,\Delta y$ is crossing the same area in the negative $x$-direction. This is equivalent to a net current flowing across the surface.

We call this current the *magnetization current* since it results from the space variation of the magnetic dipole moments induced in the magnetic material due to magnetization. The net magnetization current crossing the surface in the positive $x$-direction is

$$I_{mx} = [M_y]_{z-\Delta z/2}\,\Delta y - [M_y]_{z+\Delta z/2}\,\Delta y \tag{5.27}$$

where the subscript $m$ denotes magnetization. By dividing $I_{mx}$ by $\Delta y\,\Delta z$ and letting the area tend to zero, we obtain the magnetization current density associated with the points on the surface as

$$\begin{aligned} J_{mx} &= \lim_{\substack{\Delta y\to 0\\ \Delta z\to 0}} \frac{I_{mx}}{\Delta y\,\Delta z} = \lim_{\Delta z\to 0} \frac{[M_y]_{z-\Delta z/2} - [M_y]_{z+\Delta z/2}}{\Delta z} \\ &= -\frac{\partial M_y}{\partial z} \end{aligned} \tag{5.28}$$

or

$$J_{mx}\mathbf{a}_x = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & M_y & 0 \end{vmatrix}$$

or

$$\mathbf{J}_m = \nabla \times \mathbf{M} \tag{5.29}$$

Although we have deduced this result by considering the special case of the infinite plane current sheet, it is valid in general.

In considering electromagnetic wave propagation in a magnetic material medium, the magnetization current density given by (5.29) must be included with the current density term on the right side of Ampere's circuital law. Thus, considering Ampere's circuital law in differential form for the general case given by (3.28), we have

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J} + \mathbf{J}_m + \frac{\partial \mathbf{D}}{\partial t} \tag{5.30}$$

Substituting (5.29) into (5.30), we get

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J} + \nabla \times \mathbf{M} + \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\nabla \times \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{5.31}$$

In order to make (5.31) consistent with the corresponding equation for free space given by (3.28), we now revise the definition of the magnetic field intensity vector **H** to read as

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{5.32}$$

Substituting for **M** by using (5.25), we obtain

$$\begin{aligned} \mathbf{H} &= \frac{\mathbf{B}}{\mu_0} - \frac{\chi_m}{1 + \chi_m} \frac{\mathbf{B}}{\mu_0} \\ &= \frac{\mathbf{B}}{\mu_0(1 + \chi_m)} \\ &= \frac{\mathbf{B}}{\mu_0 \mu_r} \\ &= \frac{\mathbf{B}}{\mu} \end{aligned} \tag{5.33}$$

where we define

$$\mu_r = 1 + \chi_m \tag{5.34}$$

and

$$\mu = \mu_0 \mu_r \tag{5.35}$$

The quantity $\mu_r$ is known as the *relative permeability* of the magnetic material, and $\mu$ is the *permeability* of the magnetic material. The new definition for **H** permits the use of the same Maxwell's equations as for free space with $\mu_0$ replaced by $\mu$ and without the need for explicitly considering the magnetization current density. The permeability $\mu$ takes into account the effects of magnetization, and there is no need to consider them when we use $\mu$ for $\mu_0$! For anisotropic magnetic materials, **H** is not in general parallel to **B** and the relationship between the two quantities is expressed in the form of a matrix equation, as given by

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \tag{5.36}$$

just as in the case of the relationship between **D** and **E** for anisotropic dielectric materials.

For many materials for which the relationship between **H** and **B** is linear, the relative permeability does not differ appreciably from unity, unlike the case of linear dielectric materials, for which the relative permittivity can be very large, as shown in Table 5.2. In fact, for diamagnetic materials, the magnetic susceptibility $\chi_m$ is a small negative number of the order $-10^{-4}$ to $-10^{-8}$, whereas for paramagnetic materials, $\chi_m$ is a small positive number of the order $10^{-3}$ to $10^{-7}$. Ferromagnetic materials, however, possess large values of relative permeability on the order of several hundreds, thousands, or more. The relationship between **B** and **H** for these materials is nonlinear, resulting in a nonunique value of $\mu_r$ for a given material. In fact, these materials are characterized by hysteresis, that is, the relationship between **B** and **H** dependent on the past history of the material.

A typical curve of $B$ versus $H$, known as the *B–H curve* or the *hysteresis curve* for a ferromagnetic material, is shown in Figure 5.8. If we start with an unmagnetized sample of the material in which both $B$ and $H$ are initially zero, corresponding to point $a$ in Figure 5.8, and then magnetize the material, the manner in which magnetization is built up initially to saturation is given by the portion $ab$ of the curve. If the magnetization is now decreased gradually and then reversed in polarity, the curve does not retrace $ab$ backward but instead follows along $bcd$ until saturation is reached in the opposite direction at point $e$. A decrease in the magnetization back to zero followed by a reversal back to the original polarity brings the point back to $b$ along the curve through the points $f$ and $g$, thereby completing the loop. A continuous repetition of the process thereafter would simply make the point trace the hysteresis loop $bcdefgb$ repeatedly.

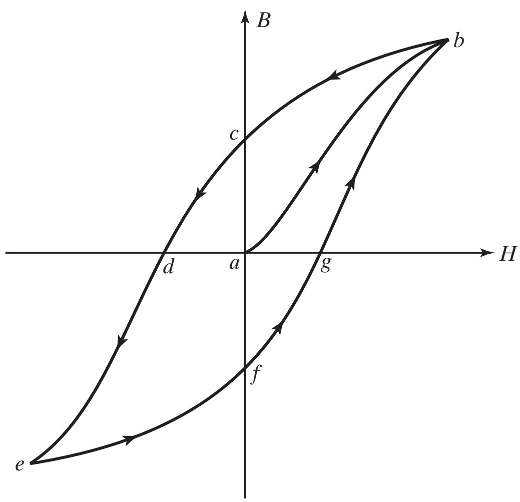


FIGURE 5.8

Hysteresis curve for a ferromagnetic material.

## 5.3 WAVE EQUATION AND SOLUTION

In the previous two sections, we introduced conductors, dielectrics, and magnetic materials. We found that conductors are characterized by conduction current, dielectrics are characterized by polarization current, and magnetic materials are characterized by magnetization current. The conduction current density is related to the electric field intensity through the conductivity $\sigma$ of the conductor. To take into account the effects of polarization, we modified the relationship between **D** and **E** by introducing the permittivity $\epsilon$ of the dielectric. Similarly, to take into account the effects of magnetization, we modified the relationship between **H** and **B** by introducing the permeability $\mu$ of the magnetic material. The three pertinent relations, known as the *constitutive relations*, are

$$\mathbf{J}_c = \sigma\mathbf{E} \tag{5.37a}$$

$$\mathbf{D} = \epsilon\mathbf{E} \tag{5.37b}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} \tag{5.37c}$$

A given material may possess all three properties, although usually one of them is predominant. Hence, in this section we shall consider a material medium characterized by $\sigma$, $\epsilon$, and $\mu$. The Maxwell's curl equations for such a medium are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu\frac{\partial \mathbf{H}}{\partial t} \tag{5.38}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma\mathbf{E} + \epsilon\frac{\partial \mathbf{E}}{\partial t} \tag{5.39}$$

To discuss electromagnetic wave propagation in the material medium, let us consider the infinite plane current sheet of Figure 4.8, except that now the medium on either side of the sheet is a material instead of free space, as shown in Figure 5.9.

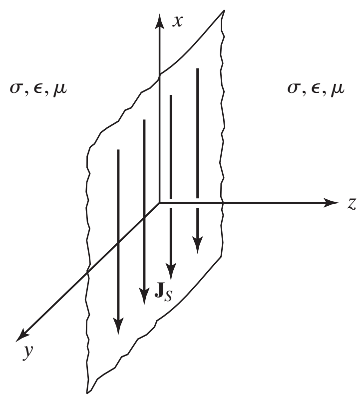


FIGURE 5.9

Infinite plane current sheet imbedded in a material medium.

The electric and magnetic fields for the simple case of the infinite plane current sheet in the $z = 0$ plane and carrying uniformly distributed current in the negative $x$-direction, as given by

$$\mathbf{J}_S = -J_{S0}\cos\omega t\,\mathbf{a}_x \tag{5.40}$$

are of the form

$$\mathbf{E} = E_x(z, t)\mathbf{a}_x \tag{5.41a}$$

$$\mathbf{H} = H_y(z, t)\mathbf{a}_y \tag{5.41b}$$

The corresponding simplified forms of the Maxwell's curl equations are

$$\frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t} \tag{5.42}$$

$$\frac{\partial H_y}{\partial z} = -\sigma E_x - \epsilon\frac{\partial E_x}{\partial t} \tag{5.43}$$

We shall make use of the phasor technique to solve these equations. Thus, letting

$$E_x(z, t) = \text{Re}\,[\bar{E}_x(z)e^{j\omega t}] \tag{5.44a}$$

$$H_y(z, t) = \text{Re}\,[\bar{H}_y(z)e^{j\omega t}] \tag{5.44b}$$

and replacing $E_x$ and $H_y$ in (5.42) and (5.43) by their phasors $\bar{E}_x$ and $\bar{H}_y$, respectively, and $\partial/\partial t$ by $j\omega$, we obtain the corresponding differential equations for the phasors $\bar{E}_x$ and $\bar{H}_y$ as

$$\frac{\partial \bar{E}_x}{\partial z} = -j\omega\mu\bar{H}_y \tag{5.45}$$

$$\frac{\partial \bar{H}_y}{\partial z} = -\sigma\bar{E}_x - j\omega\epsilon\bar{E}_x = -(\sigma + j\omega\epsilon)\bar{E}_x \tag{5.46}$$

Differentiating (5.45) with respect to $z$ and using (5.46), we obtain

$$\frac{\partial^2 \bar{E}_x}{\partial z^2} = -j\omega\mu\frac{\partial \bar{H}_y}{\partial z} = j\omega\mu(\sigma + j\omega\epsilon)\bar{E}_x \tag{5.47}$$

Defining

$$\bar{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \tag{5.48}$$

and substituting in (5.47), we have

$$\frac{\partial^2 \bar{E}_x}{\partial z^2} = \bar{\gamma}^2\bar{E}_x \tag{5.49}$$

Equation (5.49) is the wave equation for $\bar{E}_x$ in the material medium and its solution is given by

$$\bar{E}_x(z) = \bar{A}e^{-\bar{\gamma}z} + \bar{B}e^{\bar{\gamma}z} \tag{5.50}$$

where $\bar{A}$ and $\bar{B}$ are arbitrary constants. Noting that $\bar{\gamma}$ is a complex number and hence can be written as

$$\bar{\gamma} = \alpha + j\beta \tag{5.51}$$

and also writing $\bar{A}$ and $\bar{B}$ in exponential form as $Ae^{j\theta}$ and $Be^{j\phi}$, respectively, we have

$$\bar{E}_x(z) = Ae^{j\theta}e^{-\alpha z}e^{-j\beta z} + Be^{j\phi}e^{\alpha z}e^{j\beta z}$$

or

$$\begin{aligned} E_x(z,t) &= \text{Re}\,[\bar{E}_x(z)e^{j\omega t}] \\ &= \text{Re}\,[Ae^{j\theta}e^{-\alpha z}e^{-j\beta z}e^{j\omega t} + Be^{j\phi}e^{\alpha z}e^{j\beta z}e^{j\omega t}] \\ &= Ae^{-\alpha z}\cos(\omega t - \beta z + \theta) + Be^{\alpha z}\cos(\omega t + \beta z + \phi) \end{aligned} \tag{5.52}$$

We now recognize the two terms on the right side of (5.52) as representing uniform plane waves propagating in the positive $z$- and negative $z$-directions, respectively, with phase constant $\beta$, in view of the factors $\cos(\omega t - \beta z + \theta)$ and $\cos(\omega t + \beta z + \phi)$, respectively. They are, however, multiplied by the factors $e^{-\alpha z}$ and $e^{\alpha z}$, respectively. Hence, the peak amplitude of the field differs from one constant phase surface to another. Since there cannot be a positive going wave in the region $z < 0$, that is, to the left of the current sheet, and since there cannot be a negative going wave in the region $z > 0$, that is, to the right of the current sheet, the solution for the electric field is given by

$$E_x(z,t) = \begin{cases} Ae^{-\alpha z}\cos(\omega t - \beta z + \theta) & \text{for } z > 0 \\ Be^{\alpha z}\cos(\omega t + \beta z + \phi) & \text{for } z < 0 \end{cases} \tag{5.53}$$

To discuss how the peak amplitude of $E_x$ varies with $z$ on either side of the current sheet, we note that since $\sigma$, $\epsilon$, and $\mu$ are all positive, the phase angle of $j\omega\mu(\sigma + j\omega\epsilon)$ lies between 90° and 180°, and hence the phase angle of $\bar{\gamma}$ lies between 45° and 90°, making $\alpha$ and $\beta$ positive quantities. This means that $e^{-\alpha z}$ decreases with increasing value of $z$, that is, in the positive $z$-direction, and $e^{\alpha z}$ decreases with decreasing value of $z$, that is, in the negative $z$-direction. Thus, the exponential factors $e^{-\alpha z}$ and $e^{\alpha z}$ associated with the solutions for $E_x$ in (5.53) have the effect of reducing the amplitude of the field, that is, attenuating it, as it propagates away from the sheet to either side of it. For this reason, the quantity $\alpha$ is known as the *attenuation constant.* The attenuation per unit length is equal to $e^{\alpha}$. In terms of decibels, this is equal to $20\log_{10} e^{\alpha}$, or $8.686\alpha$ db. The units of $\alpha$ are nepers per meter, abbreviated Np/m. The quantity $\bar{\gamma}$ is known as the *propagation constant*, since its real and imaginary parts, $\alpha$ and $\beta$, together determine the propagation characteristics, that is, attenuation and phase shift of the wave.

Returning now to the expression for $\bar{\gamma}$ given by (5.48), we can obtain the expressions for $\alpha$ and $\beta$ by squaring it on both sides and equating the real and imaginary parts on both sides. Thus,

$$\bar{\gamma}^2 = (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

or

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \tag{5.54a}$$

$$2\alpha\beta = \omega\mu\sigma \tag{5.54b}$$

Now, squaring (5.54a) and (5.54b) and adding and then taking the square root, we obtain

$$\alpha^2 + \beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} \tag{5.55}$$

From (5.54a) and (5.55), we then have

$$\alpha^2 = \frac{1}{2}\left[-\omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\beta^2 = \frac{1}{2}\left[\omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

Since $\alpha$ and $\beta$ are both positive, we finally get

$$\alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \tag{5.56}$$

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \tag{5.57}$$

We note from (5.56) and (5.57) that $\alpha$ and $\beta$ are both dependent on $\sigma$ through the factor $\sigma/\omega\epsilon$. This factor, known as the *loss tangent*, is the ratio of the magnitude of the conduction current density $\sigma\bar{E}_x$ to the magnitude of the displacement current density $j\omega\epsilon\bar{E}_x$ in the material medium. In practice, the loss tangent is, however, not simply inversely proportional to $\omega$, since both $\sigma$ and $\epsilon$ are generally functions of frequency.

The phase velocity of the wave along the direction of propagation is given by

$$v_p = \frac{\omega}{\beta} = \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2} \tag{5.58}$$

We note that the phase velocity is dependent on the frequency of the wave. Thus, waves of different frequencies travel with different phase velocities, that is, they undergo different rates of change of phase with $z$ at any fixed time. This characteristic of the material medium gives rise to a phenomenon known as *dispersion.* The topic of dispersion is discussed in Section 8.3. The wavelength in the medium is given by

$$\lambda = \frac{2\pi}{\beta} = \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2} \tag{5.59}$$

Having found the solution for the electric field of the wave and discussed its general properties, we now turn to the solution for the corresponding magnetic field by substituting for $\bar{E}_x$ in (5.45). Thus,

$$\begin{aligned}\bar{H}_y &= -\frac{1}{j\omega\mu}\frac{\partial\bar{E}_x}{\partial z} = \frac{\bar{\gamma}}{j\omega\mu}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z}) \\ &= \sqrt{\frac{\sigma + j\omega\epsilon}{j\omega\mu}}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z}) \\ &= \frac{1}{\bar{\eta}}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z})\end{aligned} \tag{5.60}$$

where

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \tag{5.61}$$

is the intrinsic impedance of the medium. Writing

$$\bar{\eta} = |\bar{\eta}|e^{j\tau} \tag{5.62}$$

we obtain the solution for $H_y(z, t)$ as

$$\begin{aligned} H_y(z,t) &= \text{Re}\,[\bar{H}_y(z)e^{j\omega t}] \\ &= \text{Re}\left[\frac{1}{|\bar{\eta}|e^{j\tau}}Ae^{j\theta}e^{-\alpha z}e^{-j\beta z}e^{j\omega t} - \frac{1}{|\bar{\eta}|e^{j\tau}}Be^{j\phi}e^{\alpha z}e^{j\beta z}e^{j\omega t}\right] \\ &= \frac{A}{|\bar{\eta}|}e^{-\alpha z}\cos(\omega t - \beta z + \theta - \tau) - \frac{B}{|\bar{\eta}|}e^{\alpha z}\cos(\omega t + \beta z + \phi - \tau) \end{aligned} \tag{5.63}$$

Remembering that the first and second terms on the right side of (5.63) correspond to (+) and (−) waves, respectively, and hence represent the solutions for the magnetic field in the regions $z > 0$ and $z < 0$, respectively, and recalling that the solution for $H_y$ adjacent to the current sheet is given by

$$H_y = \begin{cases} \dfrac{J_{S0}}{2}\cos\omega t & \text{for } z = 0+ \\ -\dfrac{J_{S0}}{2}\cos\omega t & \text{for } z = 0- \end{cases} \tag{5.64}$$

we obtain

$$A = \frac{|\bar{\eta}|J_{S0}}{2}, \qquad \theta = \tau \tag{5.65a}$$

$$B = \frac{|\bar{\eta}|J_{S0}}{2}, \qquad \phi = \tau \tag{5.65b}$$

Thus, the electromagnetic field due to the infinite plane current sheet in the $xy$-plane having

$$\mathbf{J}_S = -J_{S0}\cos\omega t\,\mathbf{a}_x$$

and with a material medium characterized by $\sigma$, $\epsilon$, and $\mu$ on either side of it is given by

$$\mathbf{E}(z,t) = \frac{|\bar{\eta}|J_{S0}}{2}e^{\mp\alpha z}\cos(\omega t \mp \beta z + \tau)\,\mathbf{a}_x \qquad \text{for } z \gtrless 0 \tag{5.66a}$$

$$\mathbf{H}(z,t) = \pm\frac{J_{S0}}{2}e^{\mp\alpha z}\cos(\omega t \mp \beta z)\,\mathbf{a}_y \qquad \text{for } z \gtrless 0 \tag{5.66b}$$

We note from (5.66a) and (5.66b) that wave propagation in the material medium is characterized by phase difference between **E** and **H** in addition to attenuation. These properties are illustrated in Figure 5.10, which shows sketches of the current density on the sheet and the distance-variation of the electric and magnetic fields on either side of the current sheet for a few values of $t$.

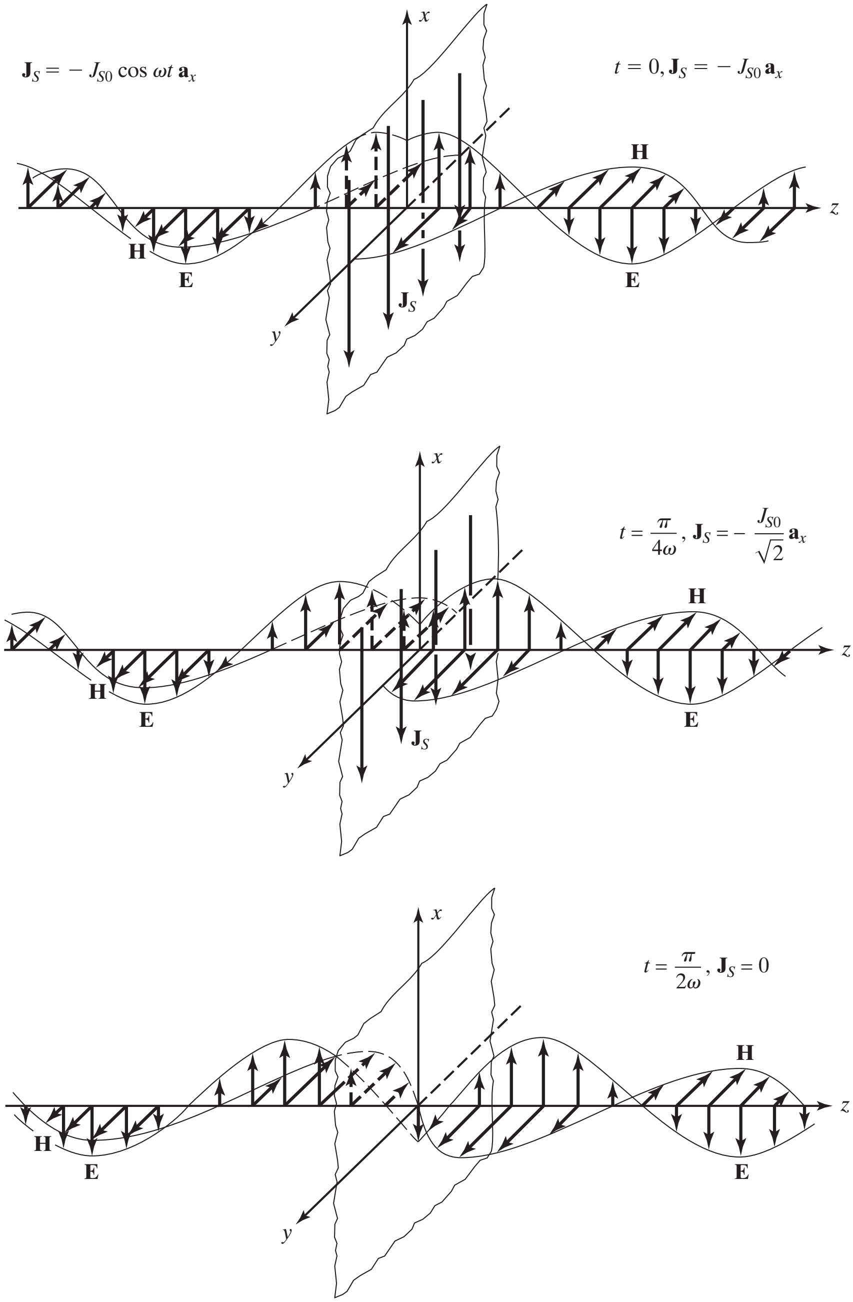


FIGURE 5.10

Time history of uniform plane electromagnetic wave radiating away from an infinite plane current sheet imbedded in a material medium.

Since the fields are attenuated as they progress in their respective directions of propagation, the medium is characterized by power dissipation. In fact, by evaluating the power flow out of a rectangular box lying between $z$ and $z + \Delta z$ and having dimensions $\Delta x$ and $\Delta y$ in the $x$- and $y$-directions, respectively, as was done in Section 4.6, we obtain

$$\begin{aligned}
\oint_S \mathbf{P} \cdot d\mathbf{S} &= \frac{\partial P_z}{\partial z}\Delta x\, \Delta y\, \Delta z = \frac{\partial}{\partial z}(E_x H_y)\,\Delta v \\
&= \left(E_x \frac{\partial H_y}{\partial z} + H_y \frac{\partial E_x}{\partial z}\right)\Delta v \\
&= \left[E_x\left(-\sigma E_x - \epsilon \frac{\partial E_x}{\partial t}\right) + H_y\left(-\mu \frac{\partial H_y}{\partial t}\right)\right]\Delta v \\
&= -\sigma E_x^2\,\Delta v - \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E_x^2\,\Delta v\right) - \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H_y^2\,\Delta v\right)
\end{aligned} \tag{5.67}$$

The quantity $\sigma E_x^2\,\Delta v$ is obviously the power dissipated in the volume $\Delta v$ due to attenuation, and the quantities $\frac{1}{2}\,\epsilon E_x^2\,\Delta v$ and $\frac{1}{2}\,\mu H_y^2\,\Delta v$ are the energies stored in the electric and magnetic fields, respectively, in the volume $\Delta v$. It then follows that the power dissipation density, the stored energy density associated with the electric field, and the stored energy density associated with the magnetic field are given by

$$P_d = \sigma E_x^2 \tag{5.68}$$

$$w_e = \frac{1}{2}\,\epsilon E_x^2 \tag{5.69}$$

and

$$w_m = \frac{1}{2}\mu H_y^2 \tag{5.70}$$

respectively. Equation (5.67) is the generalization, to the material medium, of the Poynting's theorem given by (4.70) for free space.

## 5.4 UNIFORM PLANE WAVES IN DIELECTRICS AND CONDUCTORS

In the previous section, we discussed electromagnetic wave propagation for the general case of a material medium characterized by conductivity $\sigma$, permittivity $\epsilon$, and permeability $\mu$. We found general expressions for the attenuation constant $\alpha$, the phase constant $\beta$, the phase velocity $v_p$, the wavelength $\lambda$, and the intrinsic impedance $\overline{\eta}$. These are given by (5.56), (5.57), (5.58), (5.59), and (5.61), respectively. For $\sigma = 0$, the medium is a *perfect dielectric*, having the propagation characteristics

$$\alpha = 0 \tag{5.71a}$$

$$\beta = \omega\sqrt{\mu\epsilon} \tag{5.71b}$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \tag{5.71c}$$

$$\lambda = \frac{1}{f\sqrt{\mu\epsilon}} \tag{5.71d}$$

$$\overline{\eta} = \sqrt{\frac{\mu}{\epsilon}} \tag{5.71e}$$

Thus, the waves propagate without attenuation as in free space but with $\epsilon_0$ and $\mu_0$ replaced by $\epsilon$ and $\mu$, respectively. For nonzero $\sigma$, there are two special cases: (a) imperfect dielectrics or poor conductors and (b) good conductors. The first case is characterized by conduction current small in magnitude compared to the displacement current; the second case is characterized by just the opposite.

Thus, considering the case of *imperfect dielectrics*, we have $|\sigma\overline{E}_x| \ll |j\omega\epsilon\overline{E}_x|$, or $\sigma/\omega\epsilon \ll 1$. We can then obtain approximate expressions for $\alpha$, $\beta$, $v_p$, $\lambda$, and $\overline{\eta}$ as follows:

$$\begin{aligned}
\alpha &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} \\
&= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[1+\frac{\sigma^2}{2\omega^2\epsilon^2}-\frac{\sigma^4}{8\omega^4\epsilon^4}+\cdots-1\right]^{1/2} \\
&\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\,\frac{\sigma}{\sqrt{2}\omega\epsilon}\left[1-\frac{\sigma^2}{4\omega^2\epsilon^2}\right]^{1/2} \\
&\approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72a}$$

$$\begin{aligned}
\beta &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2} \\
&\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[2+\frac{\sigma^2}{2\omega^2\epsilon^2}\right]^{1/2} \\
&\approx \omega\sqrt{\mu\epsilon}\left(1+\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72b}$$

$$\begin{aligned}
v_p &= \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[2+\frac{\sigma^2}{2\omega^2\epsilon^2}\right]^{-1/2} \\
&\approx \frac{1}{\sqrt{\mu\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72c}$$

$$\begin{aligned}
\lambda &= \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \frac{1}{f\sqrt{\mu\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72d}$$

$$\begin{aligned}\bar{\eta} &= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)^{-1/2} \\ &= \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon} - \frac{3}{8}\frac{\sigma^2}{\omega^2\epsilon^2} - \cdots\right] \\ &\approx \sqrt{\frac{\mu}{\epsilon}}\left[\left(1 - \frac{3}{8}\frac{\sigma^2}{\omega^2\epsilon^2}\right) + j\frac{\sigma}{2\omega\epsilon}\right]\end{aligned} \tag{5.72e}$$

In (5.72a)–(5.72e), we have retained all terms up to and including the second power in $\sigma/\omega\epsilon$ and have neglected all higher-order terms. For a value of $\sigma/\omega\epsilon$ equal to 0.1, the quantities $\beta, v_p$, and $\lambda$ are different from those for the corresponding perfect dielectric case by a factor of only 0.01/8, or $\frac{1}{800}$, whereas the intrinsic impedance has a real part differing from the intrinsic impedance of the perfect dielectric medium by a factor of $\frac{3}{800}$ and an imaginary part that is $\frac{1}{20}$ of the intrinsic impedance of the perfect dielectric medium. Thus, the only significant feature different from the perfect dielectric case is the attenuation.

## Example 5.2

Let us consider that a material can be classified as a dielectric for $\sigma/\omega\epsilon < 0.1$ and compute the values of the several propagation parameters for three materials: mica, dry earth, and sea water.

Denoting the frequency for which $\sigma/\omega\epsilon = 1$ as $f_q$, we have $f_q = \sigma/2\pi\epsilon$, assuming that $\sigma$ and $\epsilon$ are independent of frequency. Values of $\sigma$, $\epsilon$, and $f_q$ and approximate values of the several propagation parameters for $f > 10f_q$ are listed in Table 5.3, in which $c$ is the velocity of light in free space and $\beta_0$ and $\lambda_0$ are the phase constant and wavelength in free space for the frequency of operation. It can be seen from Table 5.3 that mica behaves as a dielectric for almost any frequency, but sea water can be classified as a dielectric only for frequencies above approximately 10 GHz. We also note that because of the low value of $\alpha$, mica is a good dielectric, but the high value of $\alpha$ for sea water makes it a poor dielectric.

TABLE 5.3 Values of Several Propagation Parameters for Three Materials for the Dielectric Range of Frequencies

| Material | $\sigma$ S/m | $\epsilon_r$ | $f_q$ Hz | $\alpha$ Np/m | $\beta/\beta_0$ | $v_p/c$ | $\lambda/\lambda_0$ | $\bar{\eta}$ $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| Mica | $10^{-11}$ | 6 | $3 \times 10^{-2}$ | $77 \times 10^{-11}$ | 2.45 | 0.408 | 0.408 | 153.9 |
| Dry earth | $10^{-5}$ | 5 | $3.6 \times 10^{4}$ | $84 \times 10^{-5}$ | 2.24 | 0.447 | 0.447 | 168.6 |
| Sea water | 4 | 80 | $0.9 \times 10^{9}$ | 84.3 | 8.94 | 0.112 | 0.112 | 42.15 |

Turning now to the case of *good conductors*, we have $|\sigma\bar{E}_x| \gg |j\omega\epsilon\bar{E}_x|$, or $\sigma/\omega\epsilon \gg 1$. We can then obtain approximate expressions for $\alpha, \beta, v_p, \lambda$, and $\eta$, as follows:

$$\begin{aligned}\alpha &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \\ &\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}} \\ &= \sqrt{\pi f\mu\sigma}\end{aligned} \tag{5.73a}$$

$$\begin{aligned}
\beta &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2} \\
&\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} \\
&= \sqrt{\pi f\mu\sigma} && (5.73b) \\
v_p &= \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\sqrt{\frac{\omega\epsilon}{\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}} \\
&= \sqrt{\frac{4\pi f}{\mu\sigma}} && (5.73c) \\
\lambda &= \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \sqrt{\frac{4\pi}{f\mu\sigma}} && (5.73d) \\
\overline{\eta} &= \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} \\
&= (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} && (5.73e)
\end{aligned}$$

We note that $\alpha, \beta, v_p$, and $\overline{\eta}$ are proportional to $\sqrt{f}$, provided that $\sigma$ and $\mu$ are constants.

To discuss the propagation characteristics of a wave inside a good conductor, let us consider the case of copper. The constants for copper are $\sigma = 5.80 \times 10^7$ S/m, $\epsilon = \epsilon_0$, and $\mu = \mu_0$. Hence, the frequency at which $\alpha$ is equal to $\omega\epsilon$ for copper is equal to $5.8 \times 10^7/2\pi\epsilon_0$, or $1.04 \times 10^{18}$ Hz. Thus, at frequencies of even several gigahertz, copper behaves like an excellent conductor. To obtain an idea of the attenuation of the wave inside the conductor, we note that the attenuation undergone in a distance of one wavelength is equal to $e^{-\alpha\lambda}$ or $e^{-2\pi}$. In terms of decibels, this is equal to $20 \log_{10} e^{2\pi} = 54.58$ db. In fact, the field is attenuated by a factor $e^{-1}$, or 0.368 in a distance equal to $1/\alpha$. This distance is known as the *skin depth* and is denoted by the symbol $\delta$. From (5.73a), we obtain

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} \tag{5.74}$$

The skin depth for copper is equal to

$$\frac{1}{\sqrt{\pi f \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}} = \frac{0.066}{\sqrt{f}} \text{ m.}$$

Thus, in copper the fields are attenuated by a factor $e^{-1}$ in a distance of 0.066 mm even at the low frequency of 1 MHz, thereby resulting in the concentration of the fields near to the skin of the conductor. This phenomenon is known as the *skin effect.* It also explains *shielding* by conductors. This topic is discussed in Section 10.3.

To discuss further the characteristics of wave propagation in a good conductor, we note that the ratio of the wavelength in the conducting medium to the wavelength in a dielectric medium having the same $\epsilon$ and $\mu$ as those of the conductor is given by

$$\frac{\lambda_{\text{conductor}}}{\lambda_{\text{dielectric}}} \approx \frac{\sqrt{4\pi/f\mu\sigma}}{1/f\sqrt{\mu\epsilon}} = \sqrt{\frac{4\pi f\epsilon}{\sigma}} = \sqrt{\frac{2\omega\epsilon}{\sigma}} \tag{5.75}$$

Since $\sigma/\omega\epsilon \gg 1$, $\lambda_{\text{conductor}} \ll \lambda_{\text{dielectric}}$. For example, for sea water, $\sigma = 4$ S/m, $\epsilon = 80\epsilon_0$, and $\mu = \mu_0$, so that the ratio of the two wavelengths for $f = 25$ kHz is equal to 0.00745. Thus for $f = 25$ kHz, the wavelength in sea water is $\frac{1}{134}$ of the wavelength in a dielectric having the same $\epsilon$ and $\mu$ as those of sea water and a still smaller fraction of the wavelength in free space. Furthermore, the lower the frequency, the smaller is this fraction. Since it is the electrical length, that is, the length in terms of the wavelength, instead of the physical length that determines the radiation efficiency of an antenna, this means that antennas of much shorter length can be used in sea water than in free space. Together with the property that $\alpha \propto \sqrt{f}$, this illustrates that low frequencies are more suitable than high frequencies for communication under water, and with underwater objects.

Equation (5.73e) tells us that the intrinsic impedance of a good conductor has a phase angle of 45º. Hence, the electric and magnetic fields in the medium are out of phase by 45º. The magnitude of the intrinsic impedance is given by

$$|\bar{\eta}| = \left|(1 + j)\sqrt{\frac{\pi f\mu}{\sigma}}\right| = \sqrt{\frac{2\pi f\mu}{\sigma}} \tag{5.76}$$

As a numerical example, for copper, this quantity is equal to

$$\sqrt{\frac{2\pi f \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 3.69 \times 10^{-7}\sqrt{f}\ \Omega$$

Thus, the intrinsic impedance of copper has as low a magnitude as 0.369 Ω even at a frequency of $10^{12}$ Hz. In fact, by recognizing that

$$|\bar{\eta}| = \sqrt{\frac{2\pi f\mu}{\sigma}} = \sqrt{\frac{\omega\epsilon}{\sigma}}\sqrt{\frac{\mu}{\epsilon}} \tag{5.77}$$

we note that the magnitude of the intrinsic impedance of a good conductor medium is a small fraction of the intrinsic impedance of a dielectric medium having the same $\epsilon$ and $\mu$. It follows that for the same electric field, the magnetic field inside a good conductor is much larger than the magnetic field inside a dielectric having the same $\epsilon$ and $\mu$ as those of the conductor.

Finally, for $\sigma = \infty$, the medium is a *perfect conductor*, an idealization of the good conductor. From (5.74), we note that the skin depth is then equal to zero and that there is no penetration of the fields. Thus, no time-varying fields can exist inside a perfect conductor.

## 5.5 BOUNDARY CONDITIONS

In our study of electromagnetics we will be considering problems involving more than one medium. To solve a problem involving a boundary surface between different media, we need to know the conditions satisfied by the field components at the boundary. These are known as the *boundary conditions.* They are a set of relationships relating the field components at a point adjacent to and on one side of the boundary, to the field components at a corresponding point adjacent to and on the other side of the boundary. These relationships arise from the fact that Maxwell's equations in integral form involve closed paths and surfaces and they must be satisfied for all possible closed paths and surfaces, whether they lie entirely in one medium or encompass a portion of the boundary between two different media. In the latter case, Maxwell's equations in integral form must be satisfied collectively by the fields on either side of the boundary, thereby resulting in the boundary conditions.

We shall derive the boundary conditions by considering the Maxwell's equations in integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S} \tag{5.78a}$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S} \tag{5.78b}$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho\, dv \tag{5.78c}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \tag{5.78d}$$

and applying them one at a time to a closed path or a closed surface encompassing the boundary, and in the limit that the area enclosed by the closed path or the volume bounded by the closed surface goes to zero. Thus, let us consider two semi-infinite media separated by a plane boundary, as shown in Figure 5.11. Let us denote the quantities pertinent to medium 1 by subscript 1 and the quantities pertinent to medium 2 by subscript 2. Let $\mathbf{a}_n$ be the unit normal vector to the surface and directed into medium 1, as shown in Figure 5.11, and let all normal components of fields at the boundary in both media denoted by an additional subscript $n$ be directed along $\mathbf{a}_n$.

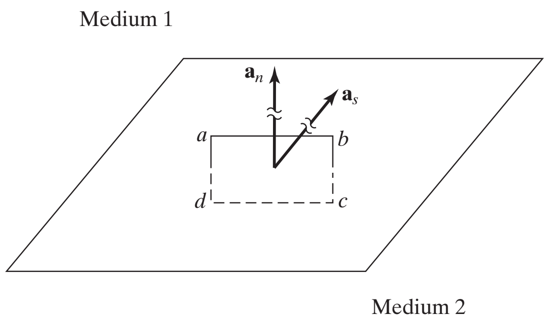


FIGURE 5.11

For deriving the boundary conditions resulting from Faraday's law and Ampere's circuital law.

Let the surface charge density ($C/m^2$) and the surface current density (A/m) on the boundary be $\rho_S$ and $\mathbf{J}_S$, respectively. Note that, in general, the fields at the boundary in both media and the surface charge and current densities are functions of position on the boundary.

First, we consider a rectangular closed path *abcda* of infinitesimal area in the plane normal to the boundary and with its sides *ab* and *cd* parallel to and on either side of the boundary, as shown in Figure 5.11. Applying Faraday's law (5.78a) to this path in the limit that *ad* and $bc \rightarrow 0$ by making the area *abcd* tend to zero, but with *ab* and *cd* remaining on either side of the boundary, we have

$$\underset{\substack{ad \to 0 \\ bc \to 0}}{\text{Lim}} \oint_{abcda} \mathbf{E} \cdot d\mathbf{l} = -\underset{\substack{ad \to 0 \\ bc \to 0}}{\text{Lim}} \frac{d}{dt} \int_{\substack{\text{area} \\ abcd}} \mathbf{B} \cdot d\mathbf{S} \tag{5.79}$$

In this limit, the contributions from *ad* and *bc* to the integral on the left side of (5.79) approach zero. Since *ab* and *cd* are infinitesimal, the sum of the contributions from *ab* and *cd* becomes $[E_{ab}(ab) + E_{cd}(cd)]$, where $E_{ab}$ and $E_{cd}$ are the components of $\mathbf{E}_1$ and $\mathbf{E}_2$ along *ab* and *cd*, respectively. The right side of (5.79) is equal to zero, since the magnetic flux crossing the area *abcd* approaches zero as the area *abcd* tends to zero. Thus, (5.79) gives

$$E_{ab}(ab) + E_{cd}(cd) = 0$$

or, since *ab* and *cd* are equal and $E_{dc} = -E_{cd}$,

$$E_{ab} - E_{dc} = 0 \tag{5.80}$$

Let us now define $\mathbf{a}_s$ to be the unit vector normal to the area *abcd* and in the direction of advance of a right-hand screw as it is turned in the sense of the closed path *abcda*. Noting then that $\mathbf{a}_s \times \mathbf{a}_n$ is the unit vector along *ab*, we can write (5.80) as

$$\mathbf{a}_s \times \mathbf{a}_n \cdot (\mathbf{E}_1 - \mathbf{E}_2) = 0$$

Rearranging the order of the scalar triple product, we obtain

$$\mathbf{a}_s \cdot \mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \tag{5.81}$$

Since we can choose the rectangle *abcd* to be in any plane normal to the boundary, (5.81) must be true for all orientations of $\mathbf{a}_s$. It then follows that

$$\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \tag{5.82a}$$

or, in scalar form,

$$E_{t1} - E_{t2} = 0 \tag{5.82b}$$

where $E_{t1}$ and $E_{t2}$ are the components of $\mathbf{E}_1$ and $\mathbf{E}_2$, respectively, tangential to the boundary. In words, (5.82a) and (5.82b) state that *at any point on the boundary, the components of* $\mathbf{E}_1$ *and* $\mathbf{E}_2$ *tangential to the boundary are equal.*

Similarly, applying Ampere's circuital law (5.78a) to the closed path in the limit that *ad* and *bc* $\rightarrow 0$, we have

$$\lim_{\substack{ad\to 0\\ bc\to 0}} \oint_{abcda} \mathbf{H}\cdot d\mathbf{l} = \lim_{\substack{ad\to 0\\ bc\to 0}} \int_{\substack{\text{area}\\ abcd}} \mathbf{J}\cdot d\mathbf{S} + \lim_{\substack{ad\to 0\\ bc\to 0}} \frac{d}{dt}\int_{\substack{\text{area}\\ abcd}} \mathbf{D}\cdot d\mathbf{S} \tag{5.83}$$

Using the same argument as for the left side of (5.79), we obtain the quantity on the left side of (5.83) to be equal to $[H_{ab}(ab) + H_{cd}(cd)]$, where $H_{ab}$ and $H_{cd}$ are the components of $\mathbf{H}_1$ and $\mathbf{H}_2$ along *ab* and *cd*, respectively. The second integral on the right side of (5.83) is zero, since the displacement flux crossing the area *abcd* approaches zero as the area *abcd* tends to zero. The first integral on the right side of (5.83) would also be equal to zero but for a contribution from the surface current on the boundary, because letting the area *abcd* tend to zero with *ab* and *cd* on either side of the boundary reduces only the volume current, if any, enclosed by it to zero, keeping the surface current still enclosed by it. This contribution is the surface current flowing normal to the line that *abcd* approaches as it tends to zero, that is, $[\mathbf{J}_S\cdot\mathbf{a}_s](ab)$. Thus, (5.83) gives

$$H_{ab}(ab) + H_{cd}(cd) = (\mathbf{J}_S\cdot\mathbf{a}_s)(ab)$$

or, since *ab* and *cd* are equal and $H_{dc} = -H_{cd}$,

$$H_{ab} - H_{dc} = \mathbf{J}_S\cdot\mathbf{a}_s \tag{5.84}$$

In terms of $\mathbf{H}_1$ and $\mathbf{H}_2$, we have

$$\mathbf{a}_s \times \mathbf{a}_n\cdot(\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S\cdot\mathbf{a}_s$$

or

$$\mathbf{a}_s\cdot\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{a}_s\cdot\mathbf{J}_S \tag{5.85}$$

Since (5.85) must be true for all orientations of $\mathbf{a}_s$, that is, for a rectangle *abcd* in any plane normal to the boundary, it follows that

$$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S \tag{5.86a}$$

or, in scalar form,

$$H_{t1} - H_{t2} = J_S \tag{5.86b}$$

where $H_{t1}$ and $H_{t2}$ are the components of $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively, tangential to the boundary. In words, (5.86a) and (5.86b) state that *at any point on the boundary, the components of* $\mathbf{H}_1$ *and* $\mathbf{H}_2$ *tangential to the boundary are discontinuous by the amount equal to the surface current density at that point.* It should be noted that the information concerning the direction of $\mathbf{J}_S$ relative to that of $(\mathbf{H}_1 - \mathbf{H}_2)$, which is contained in (5.86a), is not present in (5.86b). Thus, in general, (5.86b) is not sufficient, and it is necessary to use (5.86a).

Now, we consider a rectangular box *abcdefgh* of infinitesimal volume enclosing an infinitesimal area of the boundary and parallel to it, as shown in Figure 5.12. Applying Gauss' law for the electric field (5.78d) to this box in the limit that the side surfaces (abbreviated *ss*) tend to zero by making the volume of the box tend to zero but with the sides *abcd* and *efgh* remaining on either side of the boundary, we have

$$\lim_{ss\to 0} \oint_{\substack{\text{surface}\\ \text{of the box}}} \mathbf{D}\cdot d\mathbf{S} = \lim_{ss\to 0} \int_{\substack{\text{volume}\\ \text{of the box}}} \rho\, dv \tag{5.87}$$

In this limit, the contributions from the side surfaces to the integral on the left side of (5.87) approach zero. The sum of the contributions from the top and bottom surfaces becomes $[D_{n1}(abcd) - D_{n2}(efgh)]$, since *abcd* and *efgh* are infinitesimal. The quantity on the right side of (5.87) would be zero but for the surface charge on the boundary, since letting the volume of the box tend to zero with the sides *abcd* and *efgh* on either side of it reduces only the volume charge, if any, enclosed by it to zero, keeping the surface charge still enclosed by it. This surface charge is equal to $\rho_S(abcd)$. Thus, (5.87) gives

$$D_{n1}(abcd) - D_{n2}(efgh) = \rho_S(abcd)$$

or, since *abcd* and *efgh* are equal,

$$D_{n1} - D_{n2} = \rho_S \tag{5.88a}$$

In terms of $\mathbf{D}_1$ and $\mathbf{D}_2$, (5.88a) is given by

$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_S \tag{5.88b}$$

In words, (5.88a) and (5.88b) state that *at any point on the boundary, the components of* $\mathbf{D}_1$ *and* $\mathbf{D}_2$ *normal to the boundary are discontinuous by the amount of the surface charge density at that point.*

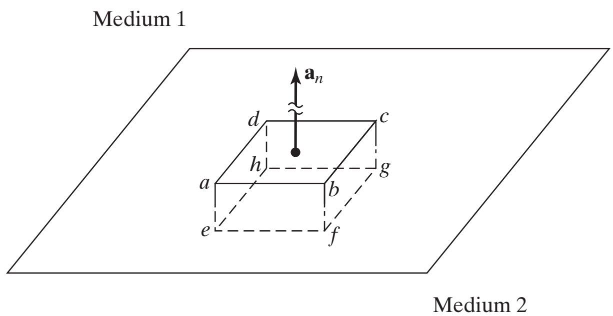


FIGURE 5.12

For deriving the boundary conditions resulting from the two Gauss' laws.

Similarly, applying Gauss' law for the magnetic field (5.78d) to the box *abcdefgh* in the limit that the side surfaces tend to zero, we have

$$\lim_{ss\to 0} \oint_{\substack{\text{surface}\\ \text{of the box}}} \mathbf{B}\cdot d\mathbf{S} = 0 \tag{5.89}$$

Using the same argument as for the left side of (5.87), we obtain the quantity on the left side of (5.89) to be equal to $[B_{n1}(abcd) - B_{n2}(efgh)]$. Thus, (5.89) gives

$$B_{n1}(abcd) - B_{n2}(efgh) = 0$$

or, since *abcd* and *efgh* are equal,

$$B_{n1} - B_{n2} = 0 \tag{5.90a}$$

In terms of $\mathbf{B}_1$ and $\mathbf{B}_2$, (5.90a) is given by

$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0 \tag{5.90b}$$

In words, (5.90a) and (5.90b) state that *at any point on the boundary, the components of* $\mathbf{B}_1$ *and* $\mathbf{B}_2$ *normal to the boundary are equal.*

Summarizing the boundary conditions, we have

$$\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \tag{5.91a}$$
$$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S \tag{5.91b}$$
$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_S \tag{5.91c}$$
$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0 \tag{5.91d}$$

or, in scalar form,

$$E_{t1} - E_{t2} = 0 \tag{5.92a}$$
$$H_{t1} - H_{t2} = J_S \tag{5.92b}$$
$$D_{n1} - D_{n2} = \rho_S \tag{5.92c}$$
$$B_{n1} - B_{n2} = 0 \tag{5.92d}$$

as illustrated in Figure 5.13. Although we have derived these boundary conditions by considering a plane interface between the two media, it should be obvious that we can consider any arbitrary-shaped boundary and obtain the same results by letting the sides *ab* and *cd* of the rectangle and the top and bottom surfaces of the box tend to zero, in addition to the limits that the sides *ad* and *bc* of the rectangle and the side surfaces of the box tend to zero.

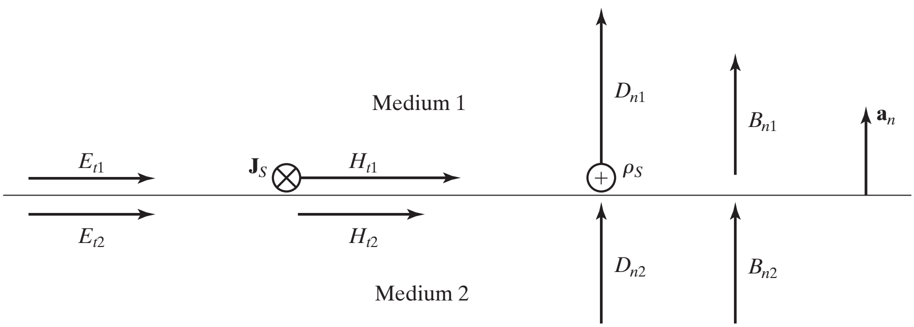


FIGURE 5.13

For illustrating the boundary conditions at an interface between two different media.

The boundary conditions given by (5.91a)–(5.91d) are general. When they are applied to particular cases, the special properties of the pertinent media come into play. Two such cases are important to be considered. They are as follows.

### Interface between Two Perfect Dielectric Media

Since for a perfect dielectric, $\sigma = 0$, $\mathbf{J}_c = \sigma\mathbf{E} = 0$. Thus, there cannot be any conduction current in a perfect dielectric, which in turn rules out any accumulation of free charge on the surface of a perfect dielectric. Hence, in applying the boundary conditions (5.91a)–(5.91d) to an interface between two perfect dielectric media, we set $\rho_S$ and $\mathbf{J}_S$ equal to zero, thereby obtaining

$$\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \tag{5.93a}$$

$$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = 0 \tag{5.93b}$$

$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = 0 \tag{5.93c}$$

$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0 \tag{5.93d}$$

These boundary conditions tell us that the tangential components of **E** and **H** and the normal components of **D** and **B** are continuous at the boundary.

### Surface of a Perfect Conductor

No time-varying fields can exist in a perfect conductor. In view of this, the boundary conditions on a perfect conductor surface are obtained by setting the fields with subscript 2 in (5.91a)–(5.91d) equal to zero. Thus, we obtain

$$\mathbf{a}_n \times \mathbf{E} = 0 \tag{5.94a}$$

$$\mathbf{a}_n \times \mathbf{H} = \mathbf{J}_S \tag{5.94b}$$

$$\mathbf{a}_n \cdot \mathbf{D} = \rho_S \tag{5.94c}$$

$$\mathbf{a}_n \cdot \mathbf{B} = 0 \tag{5.94d}$$

where we have also omitted subscripts 1, so that **E**, **H**, **D**, and **B** are the fields on the perfect conductor surface. The boundary conditions (5.94a) and (5.94d) tell us that on a perfect conductor surface, the tangential component of the electric field intensity and the normal component of the magnetic field intensity are zero. Hence, the electric field must be completely normal, and the magnetic field must be completely tangential to the surface. The remaining two boundary conditions (5.94c) and (5.94b) tell us that the (normal) displacement flux density is equal to the surface charge density and the (tangential) magnetic field intensity is equal in magnitude to the surface current density.

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### Example 5.3

In Figure 5.14, the region $x < 0$ is a perfect conductor, the region $0 < x < d$ is a perfect dielectric of $\epsilon = 2\epsilon_0$ and $\mu = \mu_0$, and the region $x > d$ is free space. The electric and magnetic fields in the region $0 < x < d$ are given at a particular instant of time by

$$\mathbf{E} = E_1 \cos \pi x \sin 2\pi z \, \mathbf{a}_x + E_2 \sin \pi x \cos 2\pi z \, \mathbf{a}_z$$

$$\mathbf{H} = H_1 \cos \pi x \sin 2\pi z \, \mathbf{a}_y$$

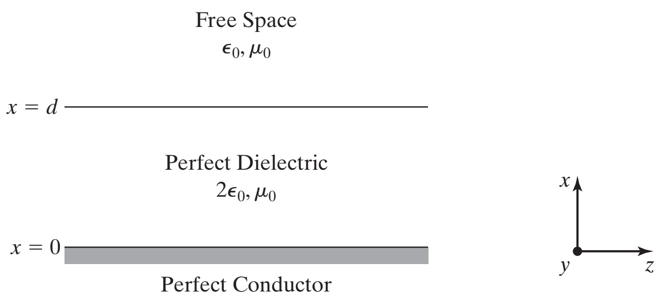


FIGURE 5.14

For illustrating the application of boundary conditions.

We wish to find (a) $\rho_S$ and $\mathbf{J}_S$ on the surface $x = 0$ and (b) $\mathbf{E}$ and $\mathbf{H}$ for $x = d+$, that is, immediately adjacent to the $x = d$ plane and on the free-space side, at that instant of time.

**(a)** Denoting the perfect dielectric medium ($0 < x < d$) to be medium 1 and the perfect conductor medium ($x < 0$) to be medium 2, we have $\mathbf{a}_n = \mathbf{a}_x$, and all fields with subscript 2 are equal to zero. Then, from (5.91c) and (5.91b), we obtain

$$\begin{aligned}[\rho_S]_{x=0} &= \mathbf{a}_n \cdot [\mathbf{D}_1]_{x=0} = \mathbf{a}_x \cdot 2\epsilon_0 E_1 \sin 2\pi z\, \mathbf{a}_x \\ &= 2\epsilon_0 E_1 \sin 2\pi z \\ [\mathbf{J}_S]_{x=0} &= \mathbf{a}_n \times [\mathbf{H}_1]_{x=0} = \mathbf{a}_x \times H_1 \sin 2\pi z\, \mathbf{a}_y \\ &= H_1 \sin 2\pi z\, \mathbf{a}_z\end{aligned}$$

Note that the remaining two boundary conditions (5.91a) and (5.91b) are already satisfied by the given fields, since $E_y$ and $B_x$ do not exist and for $x = 0$, $E_z = 0$. Also note that what we have done here is equivalent to using (5.94a)–(5.94d), since the boundary is the surface of a perfect conductor.

**(b)** Denoting the perfect dielectric medium ($0 < x < d$) to be medium 1 and the free-space medium ($x > d$) to be medium 2 and setting $\rho_S = 0$, we obtain from (5.91a) and (5.91c)

$$\begin{aligned}[E_y]_{x=d+} &= [E_y]_{x=d-} = 0 \\ [E_z]_{x=d+} &= [E_z]_{x=d-} = E_2 \sin \pi d \cos 2\pi z \\ [D_x]_{x=d+} &= [D_x]_{x=d-} = 2\epsilon_0 [E_x]_{x=d-} \\ &= 2\epsilon_0 E_1 \cos \pi d \sin 2\pi z \\ [E_x]_{x=d+} &= \frac{1}{\epsilon_0}[D_x]_{x=d+} \\ &= 2E_1 \cos \pi d \sin 2\pi z\end{aligned}$$

Thus,

$$[\mathbf{E}]_{x=d+} = 2E_1 \cos \pi d \sin 2\pi z\, \mathbf{a}_x + E_2 \sin \pi d \cos 2\pi z\, \mathbf{a}_z$$

Setting $\mathbf{J}_S = 0$ and using (5.91b) and (5.91d), we obtain

$$\begin{aligned}[H_y]_{x=d+} &= [H_y]_{x=d-} = H_1 \cos \pi d \sin 2\pi z \\ [H_z]_{x=d+} &= [H_z]_{x=d-} = 0 \\ [B_x]_{x=d+} &= [B_x]_{x=d-} = 0\end{aligned}$$

Thus,

$$[\mathbf{H}]_{x=d+} = H_1 \cos \pi d \sin 2\pi z \, \mathbf{a}_y$$

Note that what we have done here is equivalent to using (5.93a)–(5.93d), since the boundary is the interface between two perfect dielectrics.

## 5.6 REFLECTION AND TRANSMISSION OF UNIFORM PLANE WAVES

Thus far, we have considered uniform plane wave propagation in unbounded media. Practical situations are characterized by propagation involving several different media. When a wave is incident on a boundary between two different media, a reflected wave is produced. In addition, if the second medium is not a perfect conductor, a transmitted wave is set up. Together, these waves satisfy the boundary conditions at the interface between the two media. In this section, we shall consider these phenomena for waves incident normally on plane boundaries.

To do this, let us consider the situation shown in Figure 5.15 in which steady-state conditions are established by uniform plane waves of radian frequency $\omega$ propagating normal to the plane interface $z = 0$ between two media characterized by two different sets of values of $\sigma$, $\epsilon$, and $\mu$, where $\sigma \neq \infty$. We shall assume that a (+) wave is incident from medium 1 ($z < 0$) onto the interface, thereby setting up a reflected (−) wave in that medium, and a transmitted (+) wave in medium 2 ($z > 0$). For convenience, we shall work with the phasor or complex field components. Thus, considering the electric fields to be in the $x$-direction and the magnetic fields to be in the $y$-direction, we can write the solution for the complex field components in medium 1 to be

$$\bar{E}_{1x}(z) = \bar{E}_1^+ e^{-\bar{\gamma}_1 z} + \bar{E}_1^- e^{\bar{\gamma}_1 z} \tag{5.95a}$$

$$\begin{aligned} \bar{H}_{1y}(z) &= \bar{H}_1^+ e^{-\bar{\gamma}_1 z} + \bar{H}_1^- e^{\bar{\gamma}_1 z} \\ &= \frac{1}{\bar{\eta}_1}\left(\bar{E}_1^+ e^{-\bar{\gamma}_1 z} - \bar{E}_1^- e^{\bar{\gamma}_1 z}\right) \end{aligned} \tag{5.95b}$$

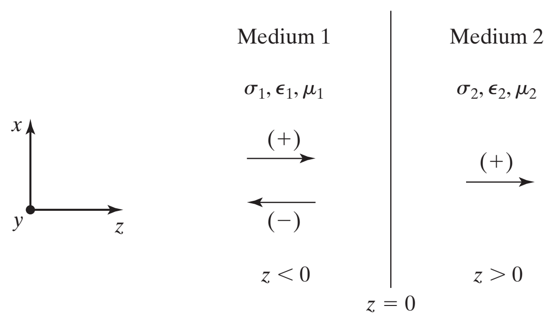


FIGURE 5.15

Normal incidence of uniform plane waves on a plane interface between two different media.

where $\bar{E}_1^+$, $\bar{E}_1^-$, $\bar{H}_1^+$, and $\bar{H}_1^-$ are the incident and reflected wave electric and magnetic field components, respectively, at $z = 0-$ in medium 1 and

$$\bar{\gamma}_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\epsilon_1)} \tag{5.96a}$$

$$\bar{\eta}_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1}} \tag{5.96b}$$

Recall that the real field corresponding to a complex field component is obtained by multiplying the complex field component by $e^{j\omega t}$ and taking the real part of the product. The complex field components in medium 2 are given by

$$\bar{E}_{2x}(z) = \bar{E}_2^+ e^{-\bar{\gamma}_2 z} \tag{5.97a}$$

$$\begin{aligned} \bar{H}_{2y}(z) &= \bar{H}_2^+ e^{-\bar{\gamma}_2 z} \\ &= \frac{\bar{E}_2^+}{\bar{\eta}_2} e^{-\bar{\gamma}_2 z} \end{aligned} \tag{5.97b}$$

where $\bar{E}_2^+$ and $\bar{H}_2^+$ are the transmitted wave electric- and magnetic-field components at $z = 0+$ in medium 2 and

$$\bar{\gamma}_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\epsilon_2)} \tag{5.98a}$$

$$\bar{\eta}_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}} \tag{5.98b}$$

To satisfy the boundary conditions at $z = 0$, we note that (1) the components of both electric and magnetic fields are entirely tangential to the interface and (2) in view of the finite conductivities of the media, no surface current exists on the interface (currents flow in the volumes of the media). Hence, from the phasor forms of the boundary conditions (5.92a) and (5.92b), we have

$$[\bar{E}_{1x}]_{z=0} = [\bar{E}_{2x}]_{z=0} \tag{5.99a}$$

$$[\bar{H}_{1y}]_{z=0} = [\bar{H}_{2y}]_{z=0} \tag{5.99b}$$

Applying these to the solution pairs given by (5.95a, b) and (5.97a, b), we have

$$\bar{E}_1^+ + \bar{E}_1^- = \bar{E}_2^+ \tag{5.100a}$$

$$\frac{1}{\bar{\eta}_1}\left(\bar{E}_1^+ - \bar{E}_1^-\right) = \frac{1}{\bar{\eta}_2}\bar{E}_2^+ \tag{5.100b}$$

We now define the *reflection coefficient* at the boundary, denoted by the symbol $\bar{\Gamma}$, to be the ratio of the reflected wave electric field at the boundary to the incident wave electric field at the boundary. From (5.100a) and (5.100b), we obtain

$$\bar{\Gamma} = \frac{\bar{E}_1^-}{\bar{E}_1^+} = \frac{\bar{\eta}_2 - \bar{\eta}_1}{\bar{\eta}_2 + \bar{\eta}_1} \tag{5.101}$$

Note that the ratio of the reflected wave magnetic field at the boundary to the incident wave magnetic field at the boundary is given by

$$\frac{\bar{H}_1^-}{\bar{H}_1^+} = \frac{-\bar{E}_1^-/\bar{\eta}_1}{\bar{E}_1^+/\bar{\eta}_1} = -\frac{\bar{E}_1^-}{\bar{E}_1^+} = -\bar{\Gamma} \tag{5.102}$$

The ratio of the transmitted wave electric field at the boundary to the incident wave electric field at the boundary, known as the *transmission coefficient* and denoted by the symbol $\bar{\tau}$, is given by

$$\bar{\tau} = \frac{\bar{E}_2^+}{\bar{E}_1^+} = \frac{\bar{E}_1^+ + \bar{E}_1^-}{\bar{E}_1^+} = 1 + \bar{\Gamma} \tag{5.103}$$

where we have used (5.100a). The ratio of the transmitted wave magnetic field at the boundary to the incident wave magnetic field at the boundary is given by

$$\frac{\bar{H}_2^+}{\bar{H}_1^+} = \frac{\bar{H}_1^+ + \bar{H}_1^-}{\bar{H}_1^+} = 1 - \bar{\Gamma} \tag{5.104}$$

The reflection and transmission coefficients given by (5.101) and (5.103), respectively, enable us to find the reflected and transmitted wave fields for a given incident wave field. We observe the following properties of $\bar{\Gamma}$ and $\bar{\tau}$:

1. For $\bar{\eta}_2 = \bar{\eta}_1$, $\bar{\Gamma} = 0$ and $\bar{\tau} = 1$. The incident wave is entirely transmitted. The situation then corresponds to a *matched* condition. A trivial case occurs when the two media have identical values of the material parameters.
2. For $\sigma_1 = \sigma_2 = 0$, that is, when both media are perfect dielectrics, $\bar{\eta}_1$ and $\bar{\eta}_2$ are real. Hence, $\bar{\Gamma}$ and $\bar{\tau}$ are real. In particular, if the two media have the same permeability $\mu$ but different permittivities $\epsilon_1$ and $\epsilon_2$, then

$$\begin{aligned}\bar{\Gamma} &= \frac{\sqrt{\mu/\epsilon_2} - \sqrt{\mu/\epsilon_1}}{\sqrt{\mu/\epsilon_2} + \sqrt{\mu/\epsilon_1}} \\ &= \frac{1 - \sqrt{\epsilon_2/\epsilon_1}}{1 + \sqrt{\epsilon_2/\epsilon_1}}\end{aligned} \tag{5.105}$$

$$\bar{\tau} = \frac{2}{1 + \sqrt{\epsilon_2/\epsilon_1}} \tag{5.106}$$

3. For $\sigma_2 \to \infty$, $\bar{\eta}_2 \to 0$, $\bar{\Gamma} \to -1$, and $\bar{\tau} \to 0$. Thus, if medium 2 is a perfect conductor, the incident wave is entirely reflected, as it should be, since there cannot be any time-varying fields inside a perfect conductor. The superposition of the reflected and incident waves would then give rise to the so-called complete standing waves in medium 1. Complete standing waves as well as partial standing waves are discussed in Chapter 7.

## Example 5.4

Region 1 ($z < 0$) is free space, whereas region 2 ($z > 0$) is a material medium characterized by $\sigma = 10^{-4}$ S/m, $\epsilon = 5\epsilon_0$, and $\mu = \mu_0$. For a uniform plane wave having the electric field

$$\mathbf{E}_i = E_0 \cos (3\pi \times 10^5 t - 10^{-3}\pi z)\, \mathbf{a}_x \text{ V/m}$$

incident on the interface $z = 0$ from region 1, we wish to obtain the expressions for the reflected and transmitted wave electric and magnetic fields.

Substituting $\sigma = 10^{-4}$ S/m, $\epsilon = 5\epsilon_0$, $\mu = \mu_0$, and $f = (3\pi \times 10^5)/2\pi = 1.5 \times 10^5$ Hz, in (5.98a) and (5.98b), we obtain

$$\bar{\gamma}_2 = (6.283 + j9.425) \times 10^{-3}$$
$$\bar{\eta}_2 = 104.559\underline{/33.69^\circ} = 104.559\underline{/0.1872\pi}$$

Then, noting that $\bar{\eta}_1 = \eta_0$,

$$\bar{\Gamma} = \frac{\bar{\eta}_2 - \eta_0}{\bar{\eta}_2 + \eta_0} = \frac{104.559\underline{/33.69^\circ} - 377}{104.559\underline{/33.69^\circ} + 377}$$
$$= 0.6325\underline{/161.565^\circ} = 0.6325\underline{/0.8976\pi}$$
$$\bar{\tau} = 1 + \bar{\Gamma} = 1 + 0.6325\underline{/161.565^\circ}$$
$$= 0.4472\underline{/26.565^\circ} = 0.4472\underline{/0.1476\pi}$$

Thus, the reflected and transmitted wave electric and magnetic fields are given by

$$\mathbf{E}_r = 0.6325E_0 \cos (3\pi \times 10^5 t + 10^{-3}\pi z + 0.8976\pi)\, \mathbf{a}_x \text{ V/m}$$
$$\mathbf{H}_r = -\frac{0.6325E_0}{377} \cos (3\pi \times 10^5 t + 10^{-3}\pi z + 0.8976\pi)\, \mathbf{a}_y \text{ A/m}$$
$$= -1.678 \times 10^{-3} E_0 \cos (3\pi \times 10^5 t + 10^{-3}\pi z + 0.8976\pi)\, \mathbf{a}_y \text{ A/m}$$
$$\mathbf{E}_t = 0.4472E_0 e^{-6.283\times10^{-3}z}$$
$$\cdot \cos (3\pi \times 10^5 t - 9.425 \times 10^{-3} z + 0.1476\pi)\, \mathbf{a}_x \text{ V/m}$$
$$\mathbf{H}_t = \frac{0.4472E_0}{104.559} e^{-6.283\times10^{-3}z}$$
$$\cdot \cos (3\pi \times 10^5 t - 9.425 \times 10^{-3} z + 0.1476\pi - 0.1872\pi)\, \mathbf{a}_y \text{ A/m}$$
$$= 4.277 \times 10^{-3} E_0 e^{-6.283\times10^{-3}z}$$
$$\cdot \cos (3\pi \times 10^5 t - 9.425 \times 10^{-3} z - 0.0396\pi)\, \mathbf{a}_y \text{ A/m}$$

Note that at $z = 0$, the boundary conditions of $\mathbf{E}_i + \mathbf{E}_r = \mathbf{E}_t$ and $\mathbf{H}_i + \mathbf{H}_r = \mathbf{H}_t$ are satisfied, since

$$E_0 + 0.6325E_0 \cos 0.8976\pi = 0.4472E_0 \cos 0.1476\pi$$

and

$$\frac{E_0}{377} - 1.678 \times 10^{-3} E_0 \cos 0.8976\pi = 4.277 \times 10^{-3} E_0 \cos (-0.0396\pi)$$

## SUMMARY

In this chapter, we studied the principles of uniform plane wave propagation in a material medium. Material media can be classified as (a) conductors, (b) dielectrics, and (c) magnetic materials, depending on the nature of the response of the charged particles in the materials to applied fields. Conductors are characterized by conduction which is the phenomenon of steady drift of free electrons under the influence of an applied electric field. Dielectrics are characterized by polarization which is the phenomenon of the creation and net alignment of electric dipoles, formed by the displacement of the centroids of the electron clouds from the centroids of the nucleii of the atoms, along the direction of an applied electric field. Magnetic materials are characterized by magnetization which is the phenomenon of net alignment of the axes of the magnetic dipoles, formed by the electron orbital and spin motion around the nucleii of the atoms, along the direction of an applied magnetic field.

Under the influence of applied electromagnetic wave fields, all three phenomena described above give rise to currents in the material which in turn influence the wave propagation. These currents are known as the conduction, polarization, and magnetization currents, respectively, for conductors, dielectrics, and magnetic materials. They must be taken into account in the first term on the right side of Ampere's circuital law, that is, $\int_S \mathbf{J} \cdot d\mathbf{S}$ in the case of the integral form and $\mathbf{J}$ in the case of the differential form. The conduction current density is given by

$$\mathbf{J}_c = \sigma\mathbf{E} \tag{5.107}$$

where $\sigma$ is the conductivity of the material. The conduction current is taken into account explicitly by replacing $\mathbf{J}$ by $\mathbf{J}_c$. The polarization and magnetization currents are taken into account implicitly by revising the definitions of the displacement flux density vector and the magnetic field intensity vector to read as

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \tag{5.108}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{5.109}$$

where $\mathbf{P}$ and $\mathbf{M}$ are the polarization and magnetization vectors, respectively. For linear isotropic materials, (5.108) and (5.109) simplify to

$$\mathbf{D} = \epsilon\mathbf{E} \tag{5.110}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} \tag{5.111}$$

where

$$\epsilon = \epsilon_0\epsilon_r$$
$$\mu = \mu_0\mu_r$$

are the permittivity and the permeability, respectively, of the material. The quantities $\epsilon_r$ and $\mu_r$ are the relative permittivity and the relative permeability, respectively, of the material. The parameters $\sigma$, $\epsilon$, and $\mu$ vary from one material to another and are in general dependent on the frequency of the wave. Equations (5.107), (5.110), and (5.111) are known as the constitutive relations. For anisotropic materials, these relations are expressed in the form of matrix equations with the material parameters represented by tensors.

Together with Maxwell's equations, the constitutive relations govern the behavior of the electromagnetic field in a material medium. Thus, Maxwell's curl equations for a material medium are given by

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu\frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma\mathbf{E} + \epsilon\frac{\partial \mathbf{E}}{\partial t}$$

We made use of these equations for the simple case of $\mathbf{E} = E_x(z, t)\mathbf{a}_x$ and $\mathbf{H} = H_y(z, t)\mathbf{a}_y$ to obtain the uniform plane wave solution by considering the infinite plane current sheet in the $xy$-plane with uniform surface current density

$$\mathbf{J}_S = -\mathbf{J}_{S0}\cos\omega t\ \mathbf{a}_x$$

and with a material medium on either side of it and finding the electromagnetic field due to the current sheet to be given by

$$\mathbf{E} = \frac{|\bar{\eta}|J_{S0}}{2}e^{\mp\alpha z}\cos(\omega t \mp \beta z + \tau)\ \mathbf{a}_x \qquad \text{for } z \gtrless 0 \tag{5.112a}$$

$$\mathbf{H} = \pm\frac{J_{S0}}{2}e^{\mp\alpha z}\cos(\omega t \mp \beta z)\ \mathbf{a}_y \qquad \text{for } z \gtrless 0 \tag{5.112b}$$

In (5.112a–b), $\alpha$ and $\beta$ are the attenuation and phase constants given, respectively, by the real and imaginary parts of the propagation constant, $\bar{\gamma}$. Thus,

$$\bar{\gamma} = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

The quantities $|\bar{\eta}|$ and $\tau$ are the magnitude and phase angle, respectively, of the intrinsic impedance, $\bar{\eta}$, of the medium. Thus,

$$\bar{\eta} = |\bar{\eta}|e^{j\tau} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

The uniform plane wave solution given by (5.112a–b) tells us that the wave propagation in the material medium is characterized by attenuation, as indicated by $e^{\mp\alpha z}$, and phase difference between **E** and **H** by the amount $\tau$. We learned that the attenuation of the wave results from power dissipation due to conduction current flow in the medium. The power dissipation density is given by

$$p_d = \sigma E_x^2$$

The stored energy densities associated with the electric and magnetic fields in the medium are given by

$$w_e = \frac{1}{2}\epsilon E^2$$
$$w_m = \frac{1}{2}\mu H^2$$

Having discussed uniform plane wave propagation for the general case of a medium characterized by $\sigma$, $\epsilon$, and $\mu$, we then considered several special cases. These are discussed in the following:

**Perfect dielectrics.** For these materials, $\sigma = 0$. Wave propagation occurs without attenuation as in free space but with the propagation parameters governed by $\epsilon$ and $\mu$ instead of $\epsilon_0$ and $\mu_0$, respectively.

**Imperfect dielectrics.** A material is classified as an imperfect dielectric for $\sigma \ll \omega\epsilon$, that is, conduction current density is small in magnitude compared to the displacement current density. The only significant feature of wave propagation in an imperfect dielectric as compared to that in a perfect dielectric is the attenuation undergone by the wave.

**Good conductors.** A material is classified as a good conductor for $\sigma \gg \omega\epsilon$, that is, conduction current density is large in magnitude compared to the displacement current density. Wave propagation in a good conductor medium is characterized by attenuation and phase constants both equal to $\sqrt{\pi f \mu\sigma}$. Thus, for large values of $f$ and/or $\sigma$, the fields do not penetrate very deeply into the conductor. This phenomenon is known as the skin effect. From considerations of the frequency dependence of the attenuation and wavelength for a fixed $\sigma$, we learned that low frequencies are more suitable for communication with underwater objects. We also learned that the intrinsic impedance of a good conductor medium is very low in magnitude compared to that of a dielectric medium having the same $\epsilon$ and $\mu$.

**Perfect conductors.** These are idealizations of good conductors in the limit $\sigma \to \infty$. For $\sigma = \infty$, the skin depth, that is, the distance in which the fields inside a conductor are attenuated by a factor $e^{-1}$, is zero and hence there can be no penetration of fields into a perfect conductor.

As a prelude to the consideration of problems involving more than one medium, we derived the boundary conditions resulting from the application of Maxwell's equations in integral form to closed paths and closed surfaces encompassing the boundary between two media, and in the limits that the areas enclosed by the closed paths and the volumes bounded by the closed surfaces go to zero. These boundary conditions are given by

$$\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$
$$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S$$
$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_S$$
$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

where the subscripts 1 and 2 refer to media 1 and 2, respectively, and $\mathbf{a}_n$ is unit vector normal to the boundary at the point under consideration and directed into medium 1. In words, the boundary conditions state that at a point on the boundary, the tangential components of **E** and the normal components of **B** are continuous, whereas the tangential components of **H** are discontinuous by the amount equal to $J_S$ at that point, and the normal components of **D** are discontinuous by the amount equal to $\rho_S$ at that point.

Two important special cases of boundary conditions are as follows: (a) At the boundary between two perfect dielectrics, the tangential components of **E** and **H** and the normal components of **D** and **B** are continuous. (b) On the surface of a perfect conductor, the tangential component of **E** and the normal component of **B** are zero, whereas the normal component of **D** is equal to the surface charge density, and the tangential component of **H** is equal in magnitude to the surface current density.

Finally, we considered uniform plane waves incident normally onto a plane boundary between two media, and we learned how to compute the reflected and transmitted wave fields for a given incident wave field.

## REVIEW QUESTIONS

**5.1.** Distinguish between bound electrons and free electrons in an atom.

**5.2.** Briefly describe the phenomenon of conduction.

**5.3.** State Ohms' law applicable at a point. How is it taken into account in Maxwell's equations?

**5.4.** Briefly describe the phenomenon of polarization in a dielectric material.

**5.5.** What is an electric dipole? How is its strength defined?

**5.6.** What are the different kinds of polarization in a dielectric?

**5.7.** What is the polarization vector? How is it related to the electric field intensity?

**5.8.** Discuss how polarization current arises in a dielectric material.

**5.9.** State the relationship between polarization current density and electric field intensity. How is it taken into account in Maxwell's equations?

**5.10.** What is the revised definition of **D**?

**5.11.** State the relationship between **D** and **E** in a dielectric material. How does it simplify the solution of field problems involving dielectrics?

**5.12.** What is an anisotropic dielectric material?

**5.13.** When can an effective permittivity be defined for an anisotropic dielectric material?

**5.14.** Briefly describe the phenomenon of magnetization.

**5.15.** What is a magnetic dipole? How is its strength defined?

**5.16.** What are the different kinds of magnetic materials?

**5.17.** What is the magnetization vector? How is it related to the magnetic flux density?

**5.18.** Discuss how magnetization current arises in a magnetic material.

**5.19.** State the relationship between magnetization current density and magnetic flux density. How is it taken into account in Maxwell's equations?

**5.20.** What is the revised definition of **H**?

**5.21.** State the relationship between **H** and **B** for a magnetic material. How does it simplify the solution of field problems involving magnetic materials?

**5.22.** What is an anisotropic magnetic material?

**5.23.** Discuss the relationship between $B$ and $H$ for a ferromagnetic material.

**5.24.** Summarize the constitutive relations for a material medium.

**5.25.** What is the propagation constant for a material medium? Discuss the significance of its real and imaginary parts.

**5.26.** Discuss the consequence of the frequency dependence of the phase velocity of a wave in a material medium.

**5.27.** What is loss tangent? Discuss its significance.

**5.28.** What is the intrinsic impedance of a material medium? What is the consequence of its complex nature?

**5.29.** How do you account for the attenuation undergone by the wave in a material medium?

**5.30.** What is the power dissipation density in a medium characterized by nonzero conductivity?

**5.31.** What are the stored energy densities associated with electric and magnetic fields in a material medium?

**5.32.** What is the condition for a medium to be a perfect dielectric? How do the characteristics of wave propagation in a perfect dielectric medium differ from those of wave propagation in free space?

**5.33.** What is the criterion for a material to be an imperfect dielectric? What is the significant feature of wave propagation in an imperfect dielectric as compared to that in a perfect dielectric?

**5.34.** Give two examples of materials that behave as good dielectrics for frequencies down to almost zero.

**5.35.** What is the criterion for a material to be a good conductor?

**5.36.** Give two examples of materials that behave as good conductors for frequencies of up to several gigahertz.

**5.37.** What is skin effect? Discuss skin depth, giving some numerical values.

**5.38.** Why are low-frequency waves more suitable than high-frequency waves for communication with underwater objects?

**5.39.** Discuss the consequence of the low intrinsic impedance of a good conductor as compared to that of a dielectric medium having the same $\epsilon$ and $\mu$.

**5.40.** Why can there be no fields inside a perfect conductor?

**5.41.** What is a boundary condition? How do boundary conditions arise and how are they derived?

**5.42.** Summarize the boundary conditions for the general case of a boundary between two arbitrary media, indicating correspondingly the Maxwell's equations in integral form from which they are derived.

**5.43.** Discuss the boundary conditions at the interface between two perfect dielectric media.

**5.44.** Discuss the boundary conditions on the surface of a perfect conductor.

**5.45.** Discuss the determination of the reflected and transitted wave fields from the fields of a wave incident normally onto a plane boundary between two material media.

**5.46.** What is the consequence of a wave incident on a perfect conductor?

## PROBLEMS

**5.1.** Find the electric field intensity required to produce a current of 0.1 A crossing an area of 1 $cm^2$ normal to the field for the following materials: (a) copper, (b) aluminum, and (c) sea water. Then find the voltage drop along a length of 1 cm parallel to the field, and find the ratio of the voltage drop to the current (resistance) for each material.

**5.2.** The free electron density in silver is $5.80 \times 10^{28}\ \text{m}^{-3}$. (a) Find the mobility of the electron for silver. (b) Find the drift velocity of the electrons for an applied electric field of intensity 0.1 V/m.

**5.3.** Use the continuity equation, Ohm's law, and Gauss' law for the electric field to show that the time variation of the charge density at a point inside a conductor is governed by the differential equation

$$\frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon_0}\rho = 0$$

Then show that the charge density inside the conductor decays exponentially with a time constant $\epsilon_0/\sigma$. Compute the value of the time constant for copper.

**5.4.** Show that the torque acting on an electric dipole of moment $\mathbf{p}$ due to an applied electric field $\mathbf{E}$ is $\mathbf{p} \times \mathbf{E}$.

**5.5.** For an applied electric field $\mathbf{E} = 0.1 \cos 2\pi \times 10^9 t\ \mathbf{a}_x$ V/m, find the polarization current crossing an area of 1 $\text{cm}^2$ normal to the field for the following materials: (a) polystyrene, (b) mica, and (c) distilled water.

**5.6.** For the anisotropic dielectric material having the permittivity tensor given in Example 5.1, find $\mathbf{D}$ for $\mathbf{E} = E_0(\cos \omega t\ \mathbf{a}_x + \sin \omega t\ \mathbf{a}_y)$. Comment on your result.

**5.7.** An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \epsilon_0 \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

(a) Find $\mathbf{D}$ for $\mathbf{E} = E_0\mathbf{a}_x$. (b) Find $\mathbf{D}$ for $\mathbf{E} = E_0(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$. (c) Find $\mathbf{E}$, which produces $\mathbf{D} = 4\epsilon_0 E_0 \mathbf{a}_x$.

**5.8.** An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

For $\mathbf{E} = (E_x\mathbf{a}_x + E_y\mathbf{a}_y) \cos \omega t$, find the value(s) of $E_y/E_x$ for which $\mathbf{D}$ is parallel to $\mathbf{E}$. Find the effective permittivity for each case.

**5.9.** Find the magnetic dipole moment of an electron in circular orbit of radius $a$ normal to a uniform magnetic field of flux density $B_0$. Compute its value for $a = 10^{-3}$ m and $B_0 = 5 \times 10^{-5}\ \text{Wb/m}^2$.

**5.10.** Show that the torque acting on a magnetic dipole of moment $\mathbf{m}$ due to an applied magnetic field $\mathbf{B}$ is $\mathbf{m} \times \mathbf{B}$. For simplicity, consider a rectangular loop in the $xy$-plane and $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$.

**5.11.** For an applied magnetic field $\mathbf{B} = 10^{-6} \cos 2\pi z\ \mathbf{a}_y\ \text{Wb/m}^2$, find the magnetization current crossing an area 1 $\text{cm}^2$ normal to the $x$-direction for a magnetic material having $\chi_m = 10^{-3}$.

**5.12.** An anisotropic magnetic material is characterized by the permeability tensor

$$[\mu] = \mu_0 \begin{bmatrix} 7 & 6 & 0 \\ 6 & 12 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find the effective permeability for $\mathbf{H} = H_0(3\mathbf{a}_x - 2\mathbf{a}_y) \cos \omega t$.

**5.13.** Obtain the wave equation for $\bar{H}_y$ similar to that for $\bar{E}_x$ given by (5.49).

**5.14.** Obtain the expression for the attenuation per wavelength undergone by a uniform plane wave in a material medium characterized by $\sigma$, $\epsilon$, and $\mu$. Using the logarithmic scale for $\sigma/\omega\epsilon$, plot the attenuation per wavelength in decibels versus $\sigma/\omega\epsilon$.

**5.15.** For dry earth, $\sigma = 10^{-5}$ S/m, $\epsilon = 5\epsilon_0$, and $\mu = \mu_0$. Compute $\alpha$, $\beta$, $v_p$, $\lambda$, and $\bar{\eta}$ for $f = 100$ kHz.

**5.16.** Obtain the expressions for the real and imaginary parts of the intrinsic impedance of a material medium given by (5.61).

**5.17.** An infinite plane sheet lying in the $xy$-plane carries current of uniform density

$$\mathbf{J}_S = -0.1 \cos 2\pi \times 10^6 t\, \mathbf{a}_x \text{ A/m}$$

The medium on either side of the sheet is characterized by $\sigma = 10^{-3}$ S/m, $\epsilon = 18\epsilon_0$, and $\mu = \mu_0$. Find $\mathbf{E}$ and $\mathbf{H}$ on either side of the current sheet.

**5.18.** Repeat Problem 5.17 for

$$\mathbf{J}_S = -0.1(\cos 2\pi \times 10^6 t\, \mathbf{a}_x + \cos 4\pi \times 10^6 t\, \mathbf{a}_x) \text{ A/m}$$

**5.19.** For an array of two infinite plane parallel current sheets of uniform densities situated in a medium characterized by $\sigma = 10^{-3}$ S/m, $\epsilon = 18\epsilon_0$, and $\mu = \mu_0$, find the spacing and the relative amplitudes and phase angles of the current densities to obtain an endfire radiation characteristic for $f = 10^6$ Hz.

**5.20.** Show that energy is not stored equally in the electric and magnetic fields in a material medium for $\sigma \neq 0$.

**5.21.** The electric field of a uniform plane wave propagating in a perfect dielectric medium having $\mu = \mu_0$ is given by

$$\mathbf{E} = 10 \cos (6\pi \times 10^7 t - 0.4\pi z)\, \mathbf{a}_x \text{ V/m}$$

Find (a) the frequency, (b) the wavelength, (c) the phase velocity, (d) the permittivity of the medium, and (e) the associated magnetic field vector $\mathbf{H}$.

**5.22.** The electric and magnetic fields of a uniform plane wave propagating in a perfect dielectric medium are given by

$$\mathbf{E} = 10 \cos (6\pi \times 10^7 t - 0.8\pi z)\, \mathbf{a}_x \text{V/m}$$

$$\mathbf{H} = \frac{1}{6\pi} \cos (6\pi \times 10^7 t - 0.8\pi z)\, \mathbf{a}_y \text{ A/m}$$

Find the permittivity and the permeability of the medium.

**5.23.** Repeat Problem 4.29 for a perfect dielectric medium of $\epsilon = 9\epsilon_0$ and $\mu = \mu_0$ on either side of the current sheet.

**5.24.** Compute $f_q$ for each of the following materials: (a) fused quartz, (b) Bakelite, and (c) distilled water. Then compute for the imperfect dielectric range of frequencies the values of $\alpha$, $\beta$, $v_p$, $\lambda$, and $\bar{\eta}$ for each material.

**5.25.** For uniform plane wave propagation in fresh water ($\sigma = 10^{-3}$ S/m, $\epsilon = 80\epsilon_0$, $\mu = \mu_0$), find $\alpha$, $\beta$, $v_p$, $\lambda$, and $\bar{\eta}$ for two frequencies: (a) 100 MHz, and (b) 10 kHz.

**5.26.** Show that for a given material, the ratio of the attenuation constant for the good conductor range of frequencies to the attenuation constant for the imperfect dielectric range of frequencies is equal to $\sqrt{2\omega\epsilon/\sigma}$ where $\omega$ is in the good conductor range of frequencies.

**5.27.** In Figure 5.16, the points 1 and 2 lie adjacent to each other and on either side of the interface between perfect dielectric media 1 and 2. The fields at point 1 are denoted by subscript 1 and the fields at point 2 are denoted by subscript 2. Assume that medium 1 is characterized by $\epsilon = 12\epsilon_0$ and $\mu = 2\mu_0$ and that medium 2 is characterized by $\epsilon = 9\epsilon_0$ and $\mu = \mu_0$. If $\mathbf{E}_1 = E_0(3\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z)$ and $\mathbf{H}_1 = H_0(2\mathbf{a}_x - 3\mathbf{a}_y)$, find $\mathbf{E}_2$ and $\mathbf{H}_2$.

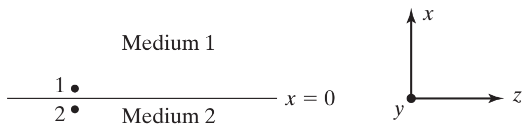


FIGURE 5.16
For Problems 5.27 and 5.28.

**5.28.** In Figure 5.16, assume that medium 1 is characterized by $\epsilon = 4\epsilon_0$ and $\mu = 3\mu_0$ and that medium 2 is characterized by $\epsilon = 16\epsilon_0$ and $\mu = 9\mu_0$. If $\mathbf{D}_1 = D_0\,(\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$ and $\mathbf{B}_1 = B_0(\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$, find $\mathbf{D}_2$ and $\mathbf{B}_2$.

**5.29.** A boundary separates free space from a perfect dielectric medium. At a point on the boundary, the electric field intensity on the free space side is $\mathbf{E}_1 = E_0(4\mathbf{a}_x + 2\mathbf{a}_y + 5\mathbf{a}_z)$, whereas on the dielectric side, it is $\mathbf{E}_2 = 3E_0(\mathbf{a}_x + \mathbf{a}_z)$, where $E_0$ is a constant. Find the permittivity of the dielectric medium.

**5.30.** The plane $x + 2y + 3z = 5$ defines the surface of a perfect conductor. Find the possible direction(s) of the electric field intensity at a point on the conductor surface.

**5.31.** Given $\mathbf{E} = y\mathbf{a}_x + x\mathbf{a}_y$, determine if a perfect conductor can be placed in the surface $xy = 2$ without disturbing the field.

**5.32.** A perfect conductor occupies the region $x + 2y \leq 2$. Find the surface current density at a point on the conductor at which $\mathbf{H} = H_0\mathbf{a}_z$.

**5.33.** The displacement flux density at a point on the surface of a perfect conductor is given by $\mathbf{D} = D_0(\mathbf{a}_x + \sqrt{3}\mathbf{a}_y + 2\sqrt{3}\mathbf{a}_z)$. Find the magnitude of the surface charge density at that point.

**5.34.** It is known that at a point on the surface of a perfect conductor $\mathbf{D} = D_0(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)$, $\mathbf{H} = H_0(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$, and $\rho_S$ is positive. Find $\rho_S$ and $\mathbf{J}_S$ at that point.

**5.35.** Two infinite plane conducting sheets occupy the planes $x = 0$ and $x = 0.1$ m. An electric field given by

$$\mathbf{E} = E_0 \sin 10\pi x \cos 3\pi \times 10^9 t\, \mathbf{a}_z$$

where $E_0$ is a constant, exists in the region between the plates, which is free space. (a) Show that $\mathbf{E}$ satisfies the boundary condition on the sheets. (b) Obtain $\mathbf{H}$ associated with the given $\mathbf{E}$. (c) Find the surface current densities on the two sheets.

**5.36.** Region $1(z < 0)$ is free space, whereas region $2(z > 0)$ is a material medium characterized by $\sigma = 10^{-3}$ S/m, $\epsilon = 12\epsilon_0$, and $\mu = \mu_0$. For a uniform plane wave having the electric field

$$\mathbf{E}_i = E_0 \cos\,(3\pi \times 10^6 t - 0.01\pi z)\,\mathbf{a}_x \text{ V/m}$$

incident on the interface $z = 0$ from region 1, obtain the expression for the reflected and transmitted wave electric and magnetic fields.

**5.37.** The regions $z < 0$ and $z > 0$ are nonmagnetic ($\mu = \mu_0$) perfect dielectrics of permittivities $\epsilon_1$ and $\epsilon_2$, respectively. For a uniform plane wave incident from the region $z < 0$ normally onto the boundary $z = 0$, find $\epsilon_2/\epsilon_1$ for each of the following to hold at $z = 0$: (a) the electric field of the reflected wave is $-1/3$ times the electric field of the incident wave; (b) the electric field of the transmitted wave is 0.4 times the electric field of the incident wave; and (c) the electric field of the transmitted wave is six times the electric field of the reflected wave.

**5.38.** A uniform plane wave propagating in the $+z$-direction and having the electric field $\mathbf{E}_i = E_{xi}(t)\mathbf{a}_x$, where $E_{xi}(t)$ in the $z = 0$ plane is as shown in Figure 5.17, is incident normally from free space ($z < 0$) onto a nonmagnetic ($\mu = \mu_0$), perfect dielectric ($z > 0$) of permittivity $4\epsilon_0$. Find and sketch the following: (a) $E_x$ versus $z$ for $t = 1\ \mu$s and (b) $H_y$ versus $z$ for $t = 1\ \mu$s.

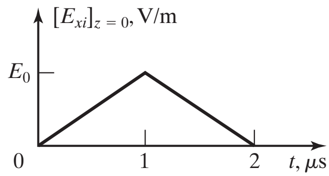


FIGURE 5.17
For Problem 5.38.

**5.39.** The region $z < 0$ is a perfect dielectric, whereas the region $z > 0$ is a perfect conductor. For a uniform plane wave having the electric and magnetic fields

$$\mathbf{E}_i = E_0 \cos(\omega t - \beta z)\, \mathbf{a}_x$$
$$\mathbf{H}_i = \frac{E_0}{\eta} \cos(\omega t - \beta z)\, \mathbf{a}_y$$

where $\beta = \omega\sqrt{\mu\epsilon}$ and $\eta = \sqrt{\mu/\epsilon}$, obtain the expressions for the reflected wave electric and magnetic fields and hence the expressions for the total (incident + reflected) electric and magnetic fields in the dielectric, and the current density on the surface of the perfect conductor.

**5.40.** In Figure 5.18, medium 3 extends to infinity so that no reflected (−) wave exists in that medium. For a uniform plane wave having the electric field

$$\mathbf{E}_i = E_0 \cos(3 \times 10^8 \pi t - \pi z)\, \mathbf{a}_x \text{ V/m}$$

incident from medium 1 onto the interface $z = 0$, obtain the expressions for the phasor electric- and magnetic-field components in all three media.

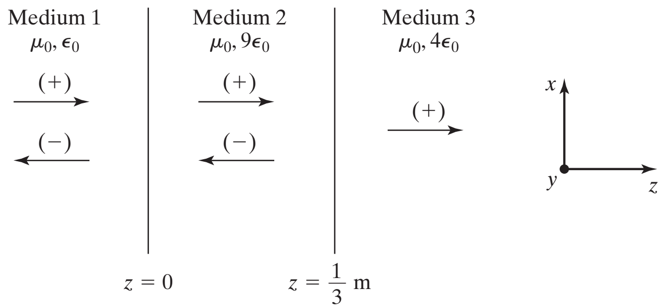


FIGURE 5.18
For Problem 5.40.