APPENDIX C

Units and Dimensions

In 1960, the International System of Units was given official status at the Eleventh General Conference on Weights and Measures held in Paris. This system of units is an expanded version of the rationalized meter–kilogram–second–ampere (MKSA) system of units and is based on six fundamental, or basic, units. The six basic units are the units of length, mass, time, current, temperature, and luminous intensity.

The international unit of length is the meter. It is exactly 1,650,763.73 times the wavelength in vacuum of the radiation corresponding to the unperturbed transition between the levels $2p_{10}$ and $5d_5$ of the atom of krypton-86, the orange-red line. The international unit of mass is the kilogram. It is the mass of the International Prototype Kilogram, which is a particular cylinder of platinum–iridium alloy preserved in a vault at Sèvres, France, by the International Bureau of Weights and Measures. The international unit of time is the second. It is equal to 9,192,631,770 times the period corresponding to the frequency of the transition between the hyperfine levels $F = 4$, $M = 0$ and $F = 3$, $M = 0$ of the fundamental state ${}^2S_{1/2}$ of the cesium-133 atom unperturbed by external fields.

To present the definition for the international unit of current, we first define the newton, which is the unit of force, derived from the fundamental units meter, kilogram, and second in the following manner. Since velocity is the rate of change of distance with time, its unit is meter per second. Since acceleration is the rate of change of velocity with time, its unit is meter per second per second, or meter per second squared. Since force is mass times acceleration, its unit is kilogram-meter per second squared, also known as the newton. Thus, the newton is that force which imparts an acceleration of 1 meter per second squared to a mass of 1 kilogram. The international unit of current, which is the ampere, can now be defined. It is the constant current that, when maintained in two straight, infinitely long, parallel conductors of negligible cross section and placed 1 meter apart in vacuum, produces a force of 2×10^{-7} newton per meter length of the conductors.

The international unit of temperature is the kelvin. It is based on the definition of the thermodynamic scale of temperature, by designating the triple point of water as a fixed fundamental point to which a temperature of exactly 273.16 kelvin is attributed. The international unit of luminous intensity is the candela. It is defined such that the luminance of a blackbody radiator at the freezing temperature of platinum is 60 candelas per square centimeter.

We have just defined the six basic units of the International System of Units. Two supplementary units are the radian and the steradian for plane angle and solid angle, respectively. All other units are derived units. For example, the unit of charge, which is the coulomb, is the amount of charge transported in 1 second by a current of 1 ampere; the unit of energy, which is the joule, is the work done when the point of application of a force of 1 newton is displaced a distance of 1 meter in the direction of the force; the unit of power, which is the watt, is the power that gives rise to the production of energy at a rate of 1 joule per second; the unit of electric potential difference, which is the volt, is the difference of electric potential between two points of a conducting wire carrying constant current of 1 ampere when the power dissipated between these points is equal to 1 watt; and so on. The units for the various quantities used in this book are listed in Table C.1, together with the symbols of the quantities and their dimensions.

TABLE C.1 Symbols, Units, and Dimensions of Various Quantities

Quantity	Symbol	Unit (Symbol)	Dimensions
Admittance	$\overline{Y}$	siemens (S)	$M^{-1}L^{-2}TQ^2$
Area	A	square meter (m^2)	L^2
Attenuation constant	α	neper/meter (Np/m)	L^{-1}
Capacitance	C	farad (F)	$M^{-1}L^{-2}T^2Q^2$
Capacitance per unit length	$\mathscr{C}$	farad/meter (F/m)	$M^{-1}L^{-3}T^2Q^2$
Cartesian coordinates	x	meter (m)	L
	y	meter (m)	L
	z	meter (m)	L
Characteristic admittance	Y_0	siemens (S)	$M^{-1}L^{-2}TQ^2$
Characteristic impedance	Z_0	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Charge	Q, q	coulomb (C)	Q
Conductance	G	siemens (S)	$M^{-1}L^{-2}TQ^2$
Conductance per unit length	$\mathscr{G}$	siemens/meter (S/m)	$M^{-1}L^{-3}TQ^2$
Conduction current density	$\mathbf{J}_c$	ampere/square meter (A/m^2)	$L^{-2}T^{-1}Q$
Conductivity	σ	siemens/meter (S/m)	$M^{-1}L^{-3}TQ^2$
Current	I	ampere (A)	$T^{-1}Q$
Cutoff frequency	f_c	hertz (Hz)	T^{-1}
Cutoff wavelength	λ_c	meter (m)	L
Cylindrical coordinates	r, r_c	meter (m)	L
	ϕ	radian	—
	z	meter (m)	L
Differential length element	$d\mathbf{l}$	meter (m)	L
Differential surface element	$d\mathbf{S}$	square meter (m^2)	L^2

(*Continued*)

TABLE C.1 (*Continued*)

Quantity	Symbol	Unit (Symbol)	Dimensions
Differential volume element	dv	cubic meter (m^3)	L^3
Directivity	D	—	—
Displacement flux density	$\mathbf{D}$	coulomb/square meter (C/m^2)	$L^{-2}Q$
Electric dipole moment	$\mathbf{p}$	coulomb-meter (C-m)	LQ
Electric field intensity	$\mathbf{E}$	volt/meter (V/m)	$MLT^{-2}Q^{-1}$
Electric potential	V	volt (V)	$ML^2T^{-2}Q^{-1}$
Electric susceptibility	χ_e	—	—
Electron density	N_e	$(\text{meter})^{-3}$ (m^{-3})	L^{-3}
Electronic charge	e	coulomb (C)	Q
Energy	W	joule (J)	ML^2T^{-2}
Energy density	w	joule/cubic meter (J/m^3)	$ML^{-1}T^{-2}$
Force	$\mathbf{F}$	newton (N)	MLT^{-2}
Frequency	f	hertz (Hz)	T^{-1}
Group velocity	v_g	meter/second (m/s)	LT^{-1}
Guide characteristic impedance	η_g	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Guide wavelength	λ_g	meter (m)	L
Impedance	$\bar{Z}$	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Inductance	L	henry (H)	ML^2Q^{-2}
Inductance per unit length	$\mathscr{L}$	henry/meter (H/m)	MLQ^{-2}
Intensity	I	watt/square meter (W/m^2)	MT^{-3}
Intrinsic impedance	η	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Length	l	meter (m)	L
Line charge density	ρ_L	coulomb/meter (C/m)	$L^{-1}Q$
Magnetic dipole moment	$\mathbf{m}$	ampere-square meter ($A\text{-}m^2$)	$L^2T^{-1}Q$
Magnetic field intensity	$\mathbf{H}$	ampere/meter (A/m)	$L^{-1}T^{-1}Q$
Magnetic flux	ψ	weber (Wb)	$ML^2T^{-1}Q^{-1}$
Magnetic flux density	$\mathbf{B}$	tesla or weber/square meter (T or Wb/m^2)	$MT^{-1}Q^{-1}$
Magnetic susceptibility	χ_m	—	—
Magnetic vector potential	$\mathbf{A}$	weber/meter (Wb/m)	$MLT^{-1}Q^{-1}$
Magnetization surface current density	$\mathbf{J}_{mS}$	ampere/meter (A/m)	$L^{-1}T^{-1}Q$
Magnetization vector	$\mathbf{M}$	ampere/meter (A/m)	$L^{-1}T^{-1}Q$
Mass	m	kilogram (kg)	M
Mobility	μ	square meter/volt-second ($m^2/V\text{-}s$)	$M^{-1}TQ$
Permeability	μ	henry/meter (H/m)	MLQ^{-2}
Permeability of free space	μ_0	henry/meter (H/m)	MLQ^{-2}
Permittivity	ε	farad/meter (F/m)	$M^{-1}L^{-3}T^2Q^2$
Permittivity of free space	ε_0	farad/meter (F/m)	$M^{-1}L^{-3}T^2Q^2$
Phase constant	β	radian/meter (rad/m)	L^{-1}
Phase velocity	v_p	meter/second (m/s)	LT^{-1}
Polarization surface charge density	ρ_{pS}	coulomb/square meter (C/m^2)	$L^{-2}Q$
Polarization vector	$\mathbf{P}$	coulomb/square meter (C/m^2)	$L^{-2}Q$

(*Continued*)

TABLE C.1 (*Continued*)

Quantity	Symbol	Unit (Symbol)	Dimensions
Power	P	watt (W)	ML^2T^{-3}
Power density	p	watt/square meter ($\mathrm{W/m^2}$)	MT^{-3}
Poynting vector	$\mathbf{P}$	watt/square meter ($\mathrm{W/m^2}$)	MT^{-3}
Propagation constant	$\bar{\gamma}$	$(\text{meter})^{-1}$ ($\mathrm{m^{-1}}$)	L^{-1}
Propagation vector	β	radian/meter (rad/m)	L^{-1}
Q factor	Q	—	—
Radian frequency	ω	radian/second (rad/s)	T^{-1}
Radiation resistance	R_{rad}	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Reactance	X	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Reflection coefficient	Γ	—	—
Refractive index	n	—	—
Relative permeability	μ_r	—	—
Relative permittivity	ε_r	—	—
Reluctance	$\mathcal{R}$	ampere (turn)/weber (A-t/Wb)	$M^{-1}L^{-2}Q^2$
Resistance	R	ohm (Ω)	$ML^2T^{-1}Q^{-2}$
Skin depth	δ	meter (m)	L
Spherical coordinates	r, r_s	meter (m)	L
	θ	radian	—
	ϕ	radian	—
Standing wave ratio	SWR	—	—
Surface charge density	ρ_S	coulomb/square meter ($\mathrm{C/m^2}$)	$L^{-2}Q$
Surface current density	$\mathbf{J}_S$	ampere/meter (A/m)	$L^{-1}T^{-1}Q$
Susceptance	B	siemens (S)	$M^{-1}L^{-2}TQ^2$
Time	t	second (s)	T
Transmission coefficient	τ	—	—
Unit normal vector	$\mathbf{a}_n$	—	—
Velocity	v	meter/second (m/s)	LT^{-1}
Velocity of light in free space	c	meter/second (m/s)	LT^{-1}
Voltage	V	volt (V)	$ML^2T^{-2}Q^{-1}$
Volume	V	cubic meter ($\mathrm{m^3}$)	L^3
Volume charge density	ρ	coulomb/cubic meter ($\mathrm{C/m^3}$)	$L^{-3}Q$
Volume current density	$\mathbf{J}$	ampere/square meter ($\mathrm{A/m^2}$)	$L^{-2}T^{-1}Q$
Wavelength	λ	meter (m)	L
Work	W	joule (J)	ML^2T^{-2}

Dimensions are a convenient means of checking the possible validity of a derived equation. The dimension of a given quantity can be expressed as some combination of a set of fundamental dimensions. These fundamental dimensions are mass (M), length (L), and time (T). In electromagnetics, it is the usual practice to consider the charge (Q), instead of the current, as the additional fundamental dimension. For the quantities listed in Table C.1, these four dimensions are sufficient. Thus, for example, the dimension of velocity is length (L) divided by time (T), that is, LT^{-1}; the dimension of acceleration is length (L) divided by time squared (T^2), that is, LT^{-2}; the dimension of force is mass (M) times acceleration (LT^{-2}), that is, MLT^{-2}; the dimension of ampere is charge (Q) divided by time (T), that is, QT^{-1}; and so on.

To illustrate the application of dimensions for checking the possible validity of a derived equation, let us consider the equation for the phase velocity of an electromagnetic wave in free space, given by

$$v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

We know that the dimension of v_p is LT^{-1}. Hence, we have to show that the dimension of $1/\sqrt{\mu_0\varepsilon_0}$ is also LT^{-1}. To do this, we note from Coulomb's law that

$$\varepsilon_0 = \frac{Q_1 Q_2}{4\pi F R^2}$$

Hence, the dimension of ε_0 is $Q^2/[(MLT^{-2})(L^2)]$, or $M^{-1}L^{-3}T^2Q^2$. We note from Ampère's law of force applied to two infinitesimal current elements parallel to each other and normal to the line joining them that

$$\mu_0 = \frac{4\pi F R^2}{(I_1\,dl_1)(I_2\,dl_2)}$$

Thus, the dimension of μ_0 is $[(MLT^{-2})(L^2)]/(QT^{-1}L)^2]$, or MLQ^{-2}. Now we obtain the dimension of $1/\sqrt{\mu_0\varepsilon_0}$ as $1/\sqrt{(M^{-1}L^{-3}T^2Q^2)(MLQ^{-2})}$, or LT^{-1}, which is the same as the dimension of v_p. It should be noted, however, that the test for the equality of the dimensions of the two sides of a derived equation is not a sufficient test to establish the equality of the two sides, since any dimensionless constants associated with the equation may be in error.

It is not always necessary to refer to the table of dimensions for checking the possible validity of a derived equation. For example, let us assume that we have derived the expression for the characteristic impedance of a transmission line (i.e., $\sqrt{\mathcal{L}/\mathcal{C}}$) and we wish to verify that $\sqrt{\mathcal{L}/\mathcal{C}}$ does indeed have the dimension of impedance. To do this, we write

$$\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{\omega\mathcal{L}l}{\omega\mathcal{C}l}} = \sqrt{\frac{\omega L}{\omega C}} = \sqrt{(\omega L)\left(\frac{1}{\omega C}\right)}$$

We now recognize from our knowledge of circuit theory that both ωL and $1/\omega C$, being the reactances of L and C, respectively, have the dimension of impedance. Hence, we conclude that $\sqrt{\mathcal{L}/\mathcal{C}}$ has the dimension of $\sqrt{(\text{impedance})^2}$ or impedance.