

Solid Mechanics Qualifying Examination Syllabus

Revised June 22, 2012

Exam Content: (will be based on material covered in, and at the level of, the following courses)

TAM 451, Intermediate Solid Mechanics

TAM 445, Continuum Mechanics

Exam Topics:

1. **Continuum Mechanics:** tensor analysis, displacement, strain, stress, conservation laws, reference frames
2. **Elasticity:** linear elastic constitutive theory, material symmetry, linear elastic boundary value problems
3. **Plasticity:** yield criteria, principle of maximum plastic resistance, plastic stability, hardening
4. **Fracture:** linear elastic fracture mechanics

Reference Textbooks:

Continuum Mechanics:

L. E. Malvern, "Introduction to the Mechanics of a Continuous Medium," Prentice Hall, (1969)

W. M. Lai, D. Rubin, and E. Krempl, "Introduction to Continuum Mechanics," Pergamon Press, (2009)

Elasticity:

L. P. Gould, "Introduction to Linear Elasticity," 2nd edition, Springer-Verlag, (1994)

A. C. Ugural and S. K. Fenster, "Advanced Strength and Applied Elasticity," 5th edition, Prentice Hall, (2011)

A. P. Boresi, K. P. Chong, and J. D. Lee, "Elasticity in Engineering Mechanics," 3rd edition, John Wiley and Sons, (2010)

Plasticity:

C. R. Calladine, "Plasticity for Engineers: Theory and Applications," Horwood Publishing Ltd, (1985)

D. C. Drucker, "Introduction to Mechanics of Deformable Solids," McGraw-Hill, (1967)

Fracture:

R. W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials," 4th edition, John Wiley & Sons, (1995)

D. Broek, "Elementary Engineering Fracture Mechanics," 4th edition, Kluwer Academic Publishers, (1982)

**QUALIFYING EXAMINATION
FOR
Solid Mechanics**

Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign

**Wednesday, August __, ____
9:00 AM – 12:00 PM**

IMPORTANT EXAMINATION INFORMATION

1. Identify your examination and work with your University Identification Number (UIN, I-Card number in blue beginning with 65) on each page. **DO NOT ENTER YOUR NAME ANYWHERE IN THE EXAMINATION.**
2. Choose 3 out of the 4 problems.
3. Each problem counts 10 points.
4. Start each problem in a new examination booklet and write on only the right-hand side (front side) of each sheet.
5. Hand in this problem package with your exam booklets.

SAMPLE EXAM FOR SOLID MECHANICS

ME PHD Qualifying Exam

Problem 1. (Elasticity)

Consider a homogeneous linear elastic body Ω with displacement components, u_i , engineering strain components, ϵ_{ij} , stress components, σ_{ij} , body force components, F_i , surface traction components, T_i , and strain energy density, $W = \frac{1}{2}\epsilon_{ij}C_{ijkl}\epsilon_{kl}$, in which the coefficients of the elasticity tensor, C_{ijkl} , exhibit the usual major and minor symmetries. Let n_i be components of the unit outward normal vector on the boundary of the body, $\partial\Omega$.

- (a) Write the engineering strain–displacement relation.
- (b) Write the local form of the governing equation for force balance.
- (c) If the force-balance equation is satisfied, what constraint does moment balance impose on the stress components?
- (d) Assume that the body is in equilibrium and that the elastic constitutive relation, $\sigma_{ij} = \partial W / \partial \epsilon_{ij}$, is satisfied. Show that $J = -2U$, in which $J := \int_{\Omega} u_i F_i d\Omega + \int_{\partial\Omega} u_i T_i dS$ and $U := \int_{\Omega} W d\Omega - J$ are the *compliance* and the *potential energy* of the body Ω .
Hint: Apply the Cauchy relation ($T_i = \sigma_{ji}n_j$), symmetry of the stress tensor, and the Divergence Theorem to the given expression for J to get started.

Problem 2. (Fracture Mechanics)

Consider a plate of thickness t comprised of a homogeneous linear elastic solid loaded under plane-stress conditions. Suppose the plate contains a crack of length $2a$ and is subjected to a far-field mode-I traction of amplitude σ . Consider also the same plate subject to the same loading but without the crack, and let U and U_0 denote the potential energies of the cracked and uncracked plates, respectively. Let the change in potential energy due to the insertion of the crack be given by

$$U - U_0 = -\frac{\pi\sigma^2 a^2 t}{E} + 4at\gamma$$

in which E is the *modulus of elasticity* and γ is the *surface energy per unit area*.

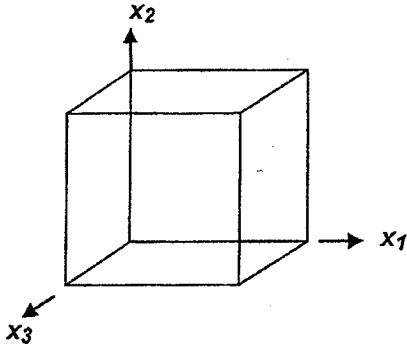
- (a) Use stationarity of the potential energy U with respect to crack length a to find the value of σ consistent with equilibrium for given values of γ , a , and E .
- (b) Show that the equilibrium condition is unstable for this system. What does this imply about the crack behavior for this theory?
- (c) What assumptions underlying the above theory make it inappropriate for use with a plate comprised of copper? Be specific.
- (d) Describe the differences between the *energy release rate*, \mathcal{G} , introduced by Irwin, and the surface energy per unit area, γ .
- (e) If the load doubles on a cracked plate whose response satisfies the small-scale yielding condition, what happens to the size of the plastic zone ahead of the crack tip?

Problem 3.

From a measurement, it is known that a cubic element of material undergoes a homogeneous deformation described by the displacement mapping

$$u_1 = \beta x_2, \quad u_2 = \alpha x_2, \quad u_3 = 0$$

where $\alpha \ll 1$ and $\beta \ll 1$. You may refer to the sketch below if it is helpful.



- 1.1 What is the deformation gradient tensor?
- 1.2 Determine the matrix of components representing the small strain tensor ϵ_{ij} corresponding to this displacement.
- 1.3 Determine the shear strain (i.e. change in angle) between the material line elements initially along $\mathbf{e}_1 + \mathbf{e}_2$ and $\mathbf{e}_1 - \mathbf{e}_2$; express your result in terms of α and β .
- 1.4 What is the volume change associated with this displacement mapping?

Problem 4.

- (a) Determine the maximum torque that can be supported by a circular cross-section shaft made of a linearly elastic-perfectly plastic material. Show the derivation from basic principles of solid mechanics.
- (b) State the Prandtl analogy for elastic torsion.
- (c) State the Nadai analogy for plastic torsion.