5. WAVE PROPAGATION IN MATERIAL MEDIA

In Chapter 4 we introduced wave propagation in free space by considering the infinite plane current sheet of uniform, sinusoidally time-varying current density. We learned that the solution for the electromagnetic field due to the infinite plane current sheet represents uniform plane electromagnetic waves propagating away from the sheet to either side of it. With the knowledge of the principles of uniform plane wave propagation in free space, we are now ready to consider wave propagation in material media, which is our goal in this chapter. Materials contain charged particles that respond to applied electric and magnetic fields and give rise to currents, which modify the properties of wave propagation from those associated with free space.

We shall learn that there are three basic phenomena resulting from the interaction of the charged particles with the electric and magnetic fields. These are conduction, polarization, and magnetization. Although a given material may exhibit all three properties, it is classified as a conductor, a dielectric, or a magnetic material depending on whether conduction, polarization, or magnetization is the predominant phenomenon. Thus we shall introduce these three kinds of materials one at a time and develop a set of relations known as the constitutive relations which enable us to avoid the necessity of explicitly taking into account the interaction of the charged particles with the fields. We shall then use these constitutive relations together with Maxwell's equations to first discuss uniform plane wave propagation in a general material medium and then consider several special cases.

5.1 CONDUCTORS

We recall that the classical model of an atom postulates a tightly bound, positively charged nucleus surrounded by a diffuse cloud of electrons spinning and orbiting around the nucleus. In the absence of an applied electromagnetic field, the force of attraction between the positively charged nucleus and the negatively charged electrons is balanced by the outward centrifugal force to maintain stable electronic orbits. The electrons can be divided into "bound" electrons and "free" or "conduction" electrons. The bound electrons can be displaced but not removed from the influence of the nucleus. The conduction electrons are constantly under thermal agitation, being released from the parent atom at one point and recaptured by another atom at a different point.

In the absence of an applied field, the motion of the conduction electrons is completely random; the average thermal velocity on a "macroscopic" scale, that is, over volumes large compared with atomic dimensions, is zero so that there is no net current and the electron cloud maintains a fixed position. With the application of an electromagnetic field, an additional velocity is superimposed on the random velocities, predominatly due to the electric force. This causes drift of the average position of the electrons in a direction opposite to that of the applied electric field. Due to the frictional mechanism provided by collisions of the electrons with the atomic lattice, the electrons, instead of accelerating under the influence of the electric field, drift with an average drift velocity proportional in magnitude to the applied electric field. This phenomenon is known as "conduction," and the resulting current due to the electron drift is known as the "conduction current."

In certain materials a large number of electrons may take part in the conduction process, but in certain other materials only a very few or negligible number of electrons may participate in conduction. The former class of materials is known as "conductors," and the latter class is known as "dielectrics" or "insulators." If the number of free electrons participating in conduction is N_e per cubic meter of the material, then the conduction current density is given by

$$\mathbf{J}_c = N_e e \mathbf{v}_d \tag{5.1}$$

where e is the charge of an electron, and $\mathbf{v}_d$ is the drift velocity of the electrons. The drift velocity varies from one conductor to another, depending on the average time between successive collisions of the electrons with the atomic lattice. It is related to the applied electric field in the manner

$$\mathbf{v}_d = -\mu_e \mathbf{E} \tag{5.2}$$

where μ_e is known as the "mobility" of the electron. Substituting (5.2) into (5.1), we obtain

$$\mathbf{J}_c = -\mu_e N_e e \mathbf{E} = \mu_e N_e |e| \mathbf{E} \tag{5.3}$$

Semiconductors are characterized by drift of "holes," that is, vacancies created by detachment of electrons from covalent bonds, in addition to the drift of electrons. If N_e and N_h are the number of electrons and holes, respectively, per cubic meter of the material and if μ_e and μ_h are the electron and hole mobilities, respectively, then the conduction current density in the semiconductor is given by

$$\mathbf{J}_c = (\mu_e N_e |e| + \mu_h N_h |e|)\mathbf{E} \tag{5.4}$$

Defining a quantity σ, known as the "conductivity" of the material, as given by

$$\sigma = \begin{cases} \mu_e N_e |e| & \text{for conductors} \\ \mu_e N_e |e| + \mu_h N_h |e| & \text{for semiconductors} \end{cases} \tag{5.5}$$

we obtain the simple and important relationship

$$\mathbf{J}_c = \sigma \mathbf{E} \tag{5.6}$$

for the conduction current density in a material. Equation (5.6) is known as Ohm's law applicable at a point from which follows the familiar form of Ohm's law used in circuit theory. The units of σ are mhos/meter where a mho ("ohm" spelled in reverse and having the symbol ℧) is an ampere per volt. Values of σ for a few materials are listed in Table 5.1. In considering electromagnetic wave propagation in conducting media, the conduction current density given by (5.6) must be employed for the current density term on the right side of Ampere's circuital law. Thus Maxwell's curl equation for **H** for

TABLE 5.1. Conductivities of Some Materials

Material	*Conductivity* mhos/m	*Material*	*Conductivity* mhos/m
Silver	6.1×10^7	Sea water	4
Copper	5.8×10^7	Intrinsic germanium	2.2
Gold	4.1×10^7	Intrinsic silicon	1.6×10^{-3}
Aluminum	3.5×10^7	Fresh water	10^{-3}
Tungsten	1.8×10^7	Distilled water	2×10^{-4}
Brass	1.5×10^7	Dry earth	10^{-5}
Solder	7.0×10^6	Bakelite	10^{-9}
Lead	4.8×10^6	Glass	10^{-10}–10^{-14}
Constantin	2.0×10^6	Mica	10^{-11}–10^{-15}
Mercury	1.0×10^6	Fused quartz	0.4×10^{-17}

a conducting medium is given by

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} \tag{5.7}$$

5.2 DIELECTRICS

In the previous section we learned that conductors are characterized by abundance of "conduction" or "free" electrons that give rise to conduction current under the influence of an applied electric field. In this section we turn our attention to dielectric materials in which the "bound" electrons are predominant. Under the application of an external electric field, the bound electrons of an atom are displaced such that the centroid of the electron cloud is separated from the centroid of the nucleus. The atom is then said to be "polarized," thereby creating an "electric dipole," as shown in Fig. 5.1(a). This kind of polarization is called "electronic polarization." The schematic representation of an electric dipole is shown in Fig. 5.1(b). The strength of the dipole is defined by the electric dipole moment $\mathbf{p}$ given by

$$\mathbf{p} = Q\mathbf{d} \tag{5.8}$$

where $\mathbf{d}$ is the vector displacement between the centroids of the positive and negative charges, each of magnitude Q coulombs.

In certain dielectric materials, polarization may exist in the molecular structure of the material even under the application of no external electric field. The polarization of individual atoms and molecules, however, is randomly oriented, and hence the net polarization on a "macroscopic" scale is zero. The application of an external field results in torques acting on the "microscopic" dipoles, as shown in Fig. 5.2, to convert the initially random polarization into a partially coherent one along the field, on a macroscopic scale. This kind of polarization is known as "orientational polarization." A third kind of polarization known as "ionic polarization" results from the separation of positive and negative ions in molecules formed by the transfer of electrons from one atom to another in the molecule. Certain materials

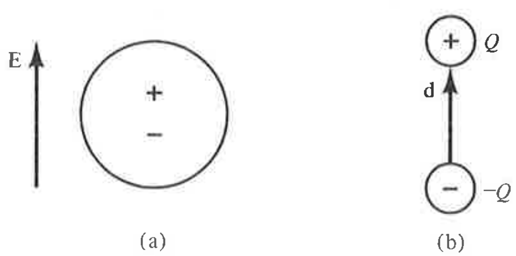

Figure 5.1. (a) An electric dipole. (b) Schematic representation of an electric dipole.

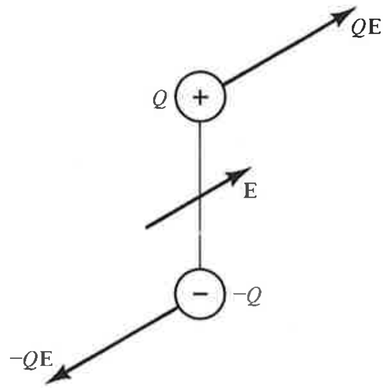

Figure 5.2. Torque acting on an electric dipole in an external electric field.

exhibit permanent polarization, that is, polarization even in the absence of an applied electric field. Electrets, when allowed to solidify in the applied electric field, become permanently polarized and ferroelectric materials exhibit spontaneous, permanent polarization.

On a macroscopic scale, we define a vector **P**, called the "polarization vector," as the "electric dipole moment per unit volume." Thus if N denotes the number of molecules per unit volume of the material, then there are $N\,\Delta v$ molecules in a volume Δv and

$$\mathbf{P} = \frac{1}{\Delta v}\sum_{j=1}^{N\Delta v}\mathbf{p}_j = N\mathbf{p} \tag{5.9}$$

where **p** is the average dipole moment per molecule. The units of **P** are coulomb-meter/meter^3 or coulombs per square meter. It is found that for many dielectric materials the polarization vector is related to the electric field **E** in the dielectric in the simple manner given by

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E} \tag{5.10}$$

where χ_e, a dimensionless parameter, is known as the "electric susceptibility." The quantity χ_e is a measure of the ability of the material to become polarized and differs from one dielectric to another.

To discuss the influence of polarization in the dielectric upon electromagnetic wave propagation in the dielectric medium, let us consider the case of the infinite plane current sheet of Fig. 4.8, radiating uniform plane waves, except that now the space on either side of the current sheet is a dielectric medium instead of being free space. The electric field in the medium induces polarization. The polarization in turn acts together with other factors to govern the behavior of the electromagnetic field. For the case under consideration, the electric field is entirely in the x direction and uniform in x and y.

Thus the induced electric dipoles are all oriented in the x direction, on a macroscopic scale, with the dipole moment per unit volume given by

$$\mathbf{P} = P_x \mathbf{i}_x = \epsilon_0 \chi_e E_x \mathbf{i}_x \tag{5.11}$$

where E_x is understood to be a function of z and t.

If we now consider an infinitesimal surface of area $\Delta y \, \Delta z$ parallel to the yz plane, we can write E_x associated with that infinitesimal area to be equal to $E_0 \cos \omega t$ where E_0 is a constant. The time history of the induced dipoles associated with that area can be sketched for one complete period of the current source, as shown in Fig. 5.3. In view of the cosinusoidal variation of

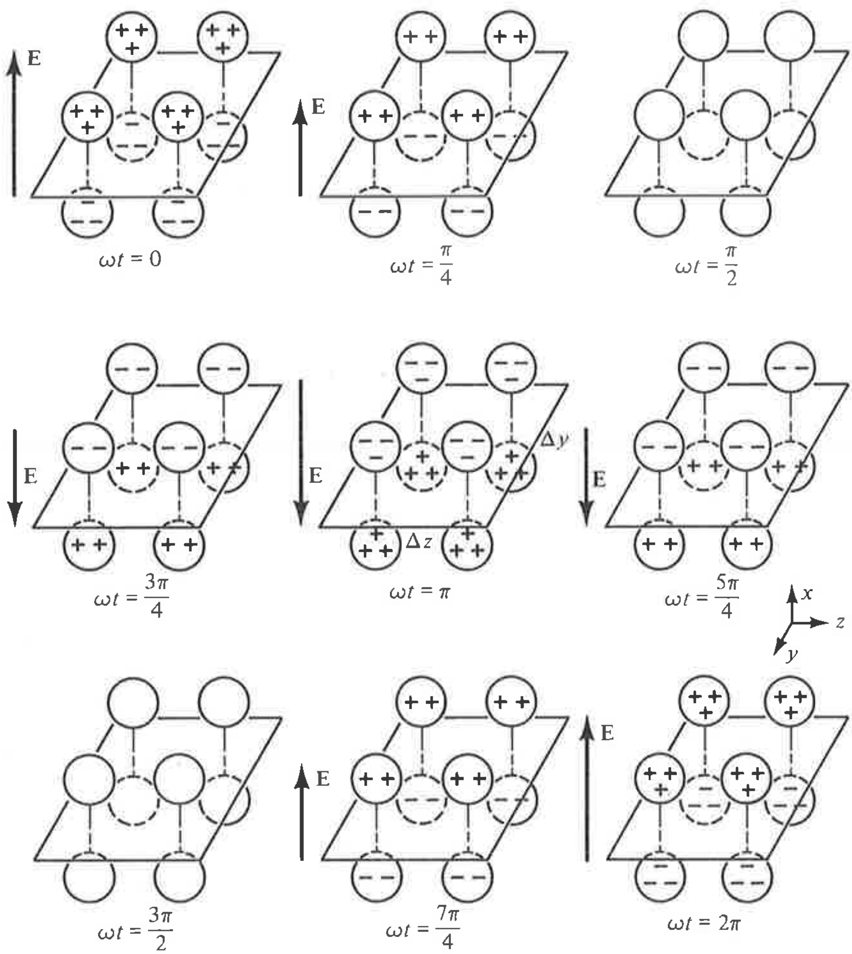

Figure 5.3. Time history of induced electric dipoles in a dielectric material under the influence of a sinusoidally time-varying electric field.

the electric field with time, the dipole moment of the individual dipoles varies in a cosinusoidal manner with maximum strength in the positive x direction at $t = 0$, decreasing sinusoidally to zero strength at $t = \pi/2\omega$ and then reversing to the negative x direction, increasing to maximum strength in that direction at $t = \pi/\omega$, and so on.

The arrangement can be considered as two plane sheets of equal and opposite time-varying charges displaced by the amount δ in the x direction, as shown in Fig. 5.4. To find the magnitude of either charge, we note that the dipole moment per unit volume is

$$P_x = \epsilon_0 \chi_e E_0 \cos \omega t \tag{5.12}$$

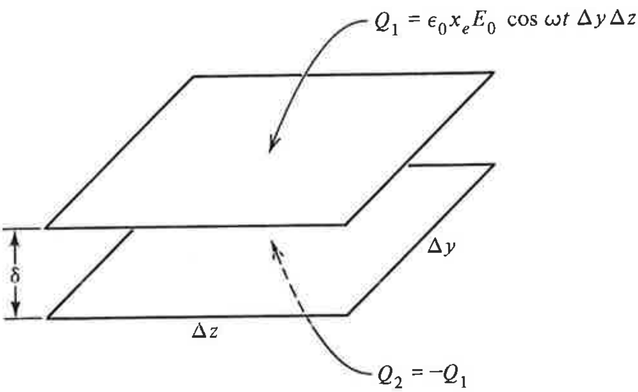

Figure 5.4. Two plane sheets of equal and opposite time-varying charges equivalent to the phenomenon depicted in Fig. 5.3.

Since the total volume occupied by the dipoles is $\delta \, \Delta y \, \Delta z$, the total dipole moment associated with the dipoles is $\epsilon_0 \chi_e E_0 \cos \omega t \, (\delta \, \Delta y \, \Delta z)$. The dipole moment associated with two equal and opposite sheet charges is equal to the magnitude of either sheet charge multiplied by the displacement between the two sheets. Hence we obtain the magnitude of either sheet charge to be $\epsilon_0 \chi_e E_0 \cos \omega t \, \Delta y \, \Delta z$. Thus we have a situation in which a sheet charge $Q_1 = \epsilon_0 \chi_e E_0 \cos \omega t \, \Delta y \, \Delta z$ is above the surface and a sheet charge $Q_2 = -Q_1 = -\epsilon_0 \chi_e E_0 \cos \omega t \, \Delta y \, \Delta z$ is below the surface. This is equivalent to a current flowing across the surface, since the charges are varying with time.

We call this current the "polarization current" since it results from the time variation of the electric dipole moments induced in the dielectric due to polarization. The polarization current crossing the surface in the positive x direction, that is, from below to above, is

$$I_{px} = \frac{dQ_1}{dt} = -\epsilon_0 \chi_e E_0 \omega \sin \omega t \, \Delta y \, \Delta z \tag{5.13}$$

where the subscript p denotes polarization. By dividing I_{px} by $\Delta y\ \Delta z$ and letting the area tend to zero, we obtain the polarization current density associated with the points on the surface as

$$J_{px} = \operatorname*{Lim}_{\substack{\Delta y \to 0 \\ \Delta z \to 0}} \frac{I_{px}}{\Delta y\ \Delta z} = -\epsilon_0 \chi_e E_0 \omega \sin \omega t$$

$$= \frac{\partial}{\partial t}(\epsilon_0 \chi_e E_0 \cos \omega t) = \frac{\partial P_x}{\partial t} \tag{5.14}$$

or

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t} \tag{5.15}$$

Although we have deduced this result by considering the special case of the infinite plane current sheet, it is valid in general.

In considering electromagnetic wave propagation in a dielectric medium, the polarization current density given by (5.15) must be included with the current density term on the right side of Ampere's circuital law. Thus considering Ampere's circuital law in differential form for the general case given by (3.28), we have

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_p + \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E}) \tag{5.16}$$

Substituting (5.15) into (5.16), we get

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{P}}{\partial t} + \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E})$$

$$= \mathbf{J} + \frac{\partial}{\partial t}(\epsilon_0\, \mathbf{E} + \mathbf{P}) \tag{5.17}$$

In order to make (5.17) consistent with the corresponding equation for free space given by (3.28), we now revise the definition of the displacement vector **D** to read as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{5.18}$$

Substituting for **P** by using (5.10), we obtain

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E}$$

$$= \epsilon_0 (1 + \chi_e) \mathbf{E}$$

$$= \epsilon_0 \epsilon_r \mathbf{E}$$

$$= \epsilon \mathbf{E} \tag{5.19}$$

where we define

$$\epsilon_r = 1 + \chi_e \tag{5.20}$$

and

$$\epsilon = \epsilon_0 \epsilon_r \tag{5.21}$$

The quantity ϵ_r is known as the "relative permittivity" or "dielectric constant" of the dielectric, and ϵ is the "permittivity" of the dielectric. The new definition for **D** permits the use of the same Maxwell's equations as for free space with ϵ_0 replaced by ϵ and without the need for explicitly considering the polarization current density. The permittivity ϵ takes into account the effects of polarization, and there is no need to consider them when we use ϵ for ϵ_0! The relative permittivity is an experimentally measurable parameter and its values for several dielectric materials are listed in Table 5.2.

TABLE 5.2. Relative Permittivities of Some Materials

Material	*Relative Permittivity*	*Material*	*Relative Permittivity*
Air	1.0006	Dry earth	5
Paper	2.0–3.0	Mica	6
Teflon	2.1	Neoprene	6.7
Polystyrene	2.56	Wet earth	10
Plexiglass	2.6–3.5	Ethyl alcohol	24.3
Nylon	3.5	Glycerol	42.5
Fused quartz	3.8	Distilled water	81
Bakelite	4.9	Titanium dioxide	100

Equation (5.19) governs the relationship between **D** and **E** for dielectric materials. Dielectrics for which ϵ is independent of the magnitude as well as the direction of **E** as indicated by (5.19) are known as "linear isotropic dielectrics." For certain dielectric materials, each component of the polarization vector can be dependent on all components of the electric field intensity. For such materials, known as "anisotropic dielectric materials," **D** is not in general parallel to **E** and the relationship between these two quantities is expressed in the form of a matrix equation as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{5.22}$$

The square matrix in (5.22) is known as the "permittivity tensor" of the anisotropic dielectric.

Example 5.1. An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \begin{bmatrix} 7\epsilon_0 & 2\epsilon_0 & 0 \\ 2\epsilon_0 & 4\epsilon_0 & 0 \\ 0 & 0 & 3\epsilon_0 \end{bmatrix}$$

Let us find **D** for several cases of **E**.

Substituting the given permittivity matrix in (5.22), we obtain

$$D_x = 7\epsilon_0 E_x + 2\epsilon_0 E_y$$
$$D_y = 2\epsilon_0 E_x + 4\epsilon_0 E_y$$
$$D_z = 3\epsilon_0 E_z$$

For $\mathbf{E} = E_0 \cos \omega t \, \mathbf{i}_z$, $\mathbf{D} = 3\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_z$; **D** is parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t \, \mathbf{i}_x$, $\mathbf{D} = 7\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_x + 2\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t \, \mathbf{i}_y$, $\mathbf{D} = 2\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_x + 4\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t \, (\mathbf{i}_x + 2\mathbf{i}_y)$, $\mathbf{D} = 11\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_x + 10\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_y$; **D** is not parallel to **E**.

For $\mathbf{E} = E_0 \cos \omega t \, (2\mathbf{i}_x + \mathbf{i}_y)$, $\mathbf{D} = 16\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_x + 8\epsilon_0 E_0 \cos \omega t \, \mathbf{i}_y = 8\epsilon_0 \mathbf{E}$; **D** is parallel to **E** and the dielectric behaves "effectively" in the same manner as an isotropic dielectric having the permittivity $8\epsilon_0$, that is, the "effective permittivity" of the anisotropic dielectric for this case is $8\epsilon_0$.

Thus we find that in general **D** is not parallel to **E** but for certain polarizations of **E**, **D** is parallel to **E**. These polarizations are known as the characteristic polarizations. ■

5.3 MAGNETIC MATERIALS

The important characteristic of magnetic materials is "magnetization." Magnetization is the phenomenon by means of which the orbital and spin motions of electrons are influenced by an external magnetic field. An electronic orbit is equivalent to a current loop, which is the magnetic analog of an electric dipole. The schematic representation of a magnetic dipole as seen from along its axis and from a point in its plane are shown in Figs. 5.5(a) and 5.5(b), respectively. The strength of the dipole is defined by the magnetic dipole moment **m** given by

$$\mathbf{m} = IA\mathbf{i}_n \tag{5.23}$$

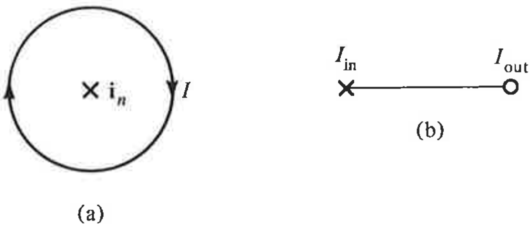

Figure 5.5. Schematic representation of a magnetic dipole as seen from (a) along its axis and (b) a point in its plane.

where A is the area enclosed by the current loop and $\mathbf{i}_n$ is the unit vector normal to the plane of the loop and directed in the right-hand sense.

In many materials the net magnetic moment of each atom is zero, that is, on the average, the magnetic dipole moments corresponding to the various electronic orbital and spin motions add up to zero. An external magnetic field has the effect of inducing a net dipole moment by changing the angular velocities of the electronic orbits, thereby magnetizing the material. This kind of magnetization, known as "diamagnetism," is in fact prevalent in all materials. In certain materials known as "paramagnetic materials," the individual atoms possess net nonzero magnetic moments even in the absence of an external magnetic field. These "permanent" magnetic moments of the individual atoms are, however, randomly oriented so that the net magnetization on a macroscopic scale is zero. An applied magnetic field has the effect of exerting torques on the individual permanent dipoles as shown in Fig. 5.6 to convert, on a macroscopic scale, the initially random alignment into a partially coherent one along the magnetic field, that is, with the normal to the current loop directed along the magnetic field. This kind of magnetization is known as "paramagnetism." Certain materials known as "ferromagnetic," "antiferromagnetic," and "ferrimagnetic" materials exhibit permanent magnetization, that is, magnetization even in the absence of an applied magnetic field.

On a macroscopic scale we define a vector **M**, called the "magnetization

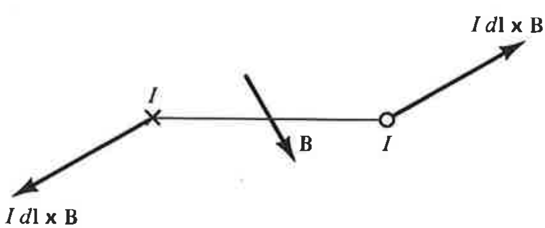

Figure 5.6. Torque acting on a magnetic dipole in an external magnetic field.

vector," as the "magnetic dipole moment per unit volume." Thus if N denotes the number of molecules per unit volume of the material, then there are $N\,\Delta v$ molecules in a volume Δv and

$$\mathbf{M} = \frac{1}{\Delta v}\sum_{j=1}^{N\,\Delta v} \mathbf{m}_j = N\mathbf{m} \tag{5.24}$$

where $\mathbf{m}$ is the average dipole moment per molecule. The units of $\mathbf{M}$ are ampere-meter²/meter³ or amperes per meter. It is found that for many magnetic materials, the magnetization vector is related to the magnetic field $\mathbf{B}$ in the material in the simple manner given by

$$\mathbf{M} = \frac{\chi_m}{1+\chi_m}\frac{\mathbf{B}}{\mu_0} \tag{5.25}$$

where χ_m, a dimensionless parameter, is known as the "magnetic susceptibility." The quantity χ_m is a measure of the ability of the material to become magnetized and differs from one magnetic material to another.

To discuss the influence of magnetization in the material on electromagnetic wave propagation in the magnetic material medium, let us consider the case of the infinite plane current sheet of Fig. 4.8, radiating uniform plane waves, except that now the space on either side of the current sheet possesses magnetic material properties in addition to dielectric properties. The magnetic field in the medium induces magnetization. The magnetization in turn acts together with other factors to govern the behavior of the electromagnetic field. For the case under consideration, the magnetic field is entirely in the y direction and uniform in x and y. Thus the induced dipoles are all oriented with their axes in the y direction, on a macroscopic scale, with the dipole moment per unit volume given by

$$\mathbf{M} = M_y\mathbf{i}_y = \frac{\chi_m}{1+\chi_m}\frac{B_y}{\mu_0}\mathbf{i}_y \tag{5.26}$$

where B_y is understood to be a function of z and t.

Let us now consider an infinitesimal surface of area $\Delta y\,\Delta z$ parallel to the yz plane and the magnetic dipoles associated with the two areas $\Delta y\,\Delta z$ to the left and to the right of the center of this area as shown in Fig. 5.7(a). Since B_y is a function of z, we can assume the dipoles in the left area to have a different moment than the dipoles in the right area for any given time. If the dimension of an individual dipole is δ in the x direction, then the total dipole moment associated with the dipoles in the left area is $[M_y]_{z-\Delta z/2}\,\delta\,\Delta y\,\Delta z$ and the total dipole moment associated with the dipoles in the right area is $[M_y]_{z+\Delta z/2}\,\delta\,\Delta y\,\Delta z$.

The arrangement of dipoles can be considered to be equivalent to two rectangular surface current loops as shown in Fig. 5.7(b) with the left side

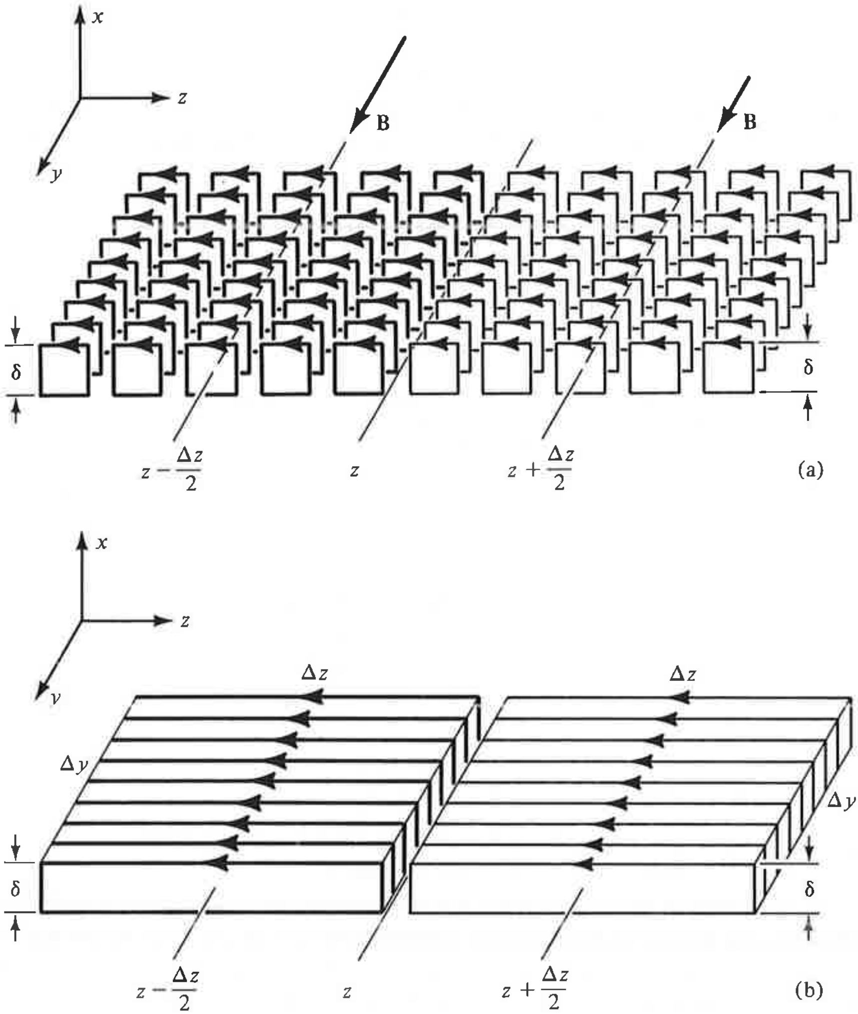

Figure 5.7. (a) Induced magnetic dipoles in a magnetic material. (b) Equivalent surface current loops.

current loop having a dipole moment $[M_y]_{z-\Delta z/2}\, \delta\, \Delta y\, \Delta z$ and the right side current loop having a dipole moment $[M_y]_{z+\Delta z/2}\, \delta\, \Delta y\, \Delta z$. Since the magnetic dipole moment of a rectangular surface current loop is simply equal to the product of the surface current and the cross-sectional area of the loop, the surface current associated with the left loop is $[M_y]_{z-\Delta z/2}\, \Delta y$ and the surface current associated with the right loop is $[M_y]_{z+\Delta z/2}\, \Delta y$. Thus we have a stituation in which a current equal to $[M_y]_{z-\Delta z/2}\, \Delta y$ is crossing the area $\Delta y\, \Delta z$ in the positive x direction, and a current equal to $[M_y]_{z+\Delta z/2}\, \Delta y$ is crossing the same area in the negative x direction. This is equivalent to a net current flowing across the surface.

We call this current the "magnetization current" since it results from the space variation of the magnetic dipole moments induced in the magnetic material due to magnetization. The net magnetization current crossing the surface in the positive x direction is

$$I_{mx} = [M_y]_{z-\Delta z/2}\,\Delta y - [M_y]_{z+\Delta z/2}\,\Delta y \tag{5.27}$$

where the subscript m denotes magnetization. By dividing I_{mx} by $\Delta y\,\Delta z$ and letting the area tend to zero, we obtain the magnetization current density associated with the points on the surface as

$$\begin{aligned} J_{mx} &= \lim_{\substack{\Delta y \to 0 \\ \Delta z \to 0}} \frac{I_{mx}}{\Delta y\,\Delta z} = \lim_{\Delta z \to 0} \frac{[M_y]_{z-\Delta z/2} - [M_y]_{z+\Delta z/2}}{\Delta z} \\ &= -\frac{\partial M_y}{\partial z} \end{aligned} \tag{5.28}$$

or

$$J_{mx}\mathbf{i}_x = \begin{vmatrix} \mathbf{i}_x & \mathbf{i}_y & \mathbf{i}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & M_y & 0 \end{vmatrix}$$

or

$$\mathbf{J}_m = \nabla \times \mathbf{M} \tag{5.29}$$

Although we have deduced this result by considering the special case of the infinite plane current sheet, it is valid in general.

In considering electromagnetic wave propagation in a magnetic material medium, the magnetization current density given by (5.29) must be included with the current density term on the right side of Ampere's circuital law. Thus considering Ampere's circuital law in differential form for the general case given by (3.28), we have

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J} + \mathbf{J}_m + \frac{\partial \mathbf{D}}{\partial t} \tag{5.30}$$

Substituting (5.29) into (5.30), we get

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J} + \nabla \times \mathbf{M} + \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{5.31}$$

In order to make (5.31) consistent with the corresponding equation for free space given by (3.28), we now revise the definition of the magnetic field intensity vector **H** to read as

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{5.32}$$

Substituting for **M** by using (5.25), we obtain

$$\begin{aligned}\mathbf{H} &= \frac{\mathbf{B}}{\mu_0} - \frac{\chi_m}{1+\chi_m}\frac{\mathbf{B}}{\mu_0} \\ &= \frac{\mathbf{B}}{\mu_0(1+\chi_m)} \\ &= \frac{\mathbf{B}}{\mu_0\mu_r} \\ &= \frac{\mathbf{B}}{\mu}\end{aligned} \tag{5.33}$$

where we define

$$\mu_r = 1 + \chi_m \tag{5.34}$$

and

$$\mu = \mu_0\mu_r \tag{5.35}$$

The quantity μ_r is known as the "relative permeability" of the magnetic material and μ is the "permeability" of the magnetic material. The new definition for **H** permits the use of the same Maxwell's equations as for free space with μ_0 replaced by μ and without the need for explicitly considering the magnetization current density. The permeability μ takes into account the effects of magnetization, and there is no need to consider them when we use μ for μ_0! For anisotropic magnetic materials, **H** is not in general parallel to **B** and the relationship between the two quantities is expressed in the form of a matrix equation as given by

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \tag{5.36}$$

just as in the case of the relationship between **D** and **E** for anisotropic dielectric materials.

For many materials for which the relationship between **H** and **B** is linear, the relative permeability does not differ appreciably from unity, unlike the case of linear dielectric materials, for which the relative permittivity can be very large, as shown in Table 5.2. In fact, for diamagnetic materials, the magnetic susceptibility χ_m is a small negative number of the order -10^{-4} to -10^{-8} whereas for paramagnetic materials, χ_m is a small positive number of the order 10^{-3} to 10^{-7}. Ferromagnetic materials, however, possess large values of relative permeability on the order of several hundreds, thousands, or more. The relationship between **B** and **H** for these materials is nonlinear, resulting in a nonunique value of μ_r for a given material. In fact, these materials are characterized by hysteresis, that is, the relationship between **B** and **H** dependent on the past history of the material.

A typical curve of B versus H, known as the "B–H curve" or the "hysteresis curve" for a ferromagnetic material, is shown in Fig. 5.8. If we start with an unmagnetized sample of the material in which both B and H are initially zero, corresponding to point a in Fig. 5.8, and then magnetize the material,

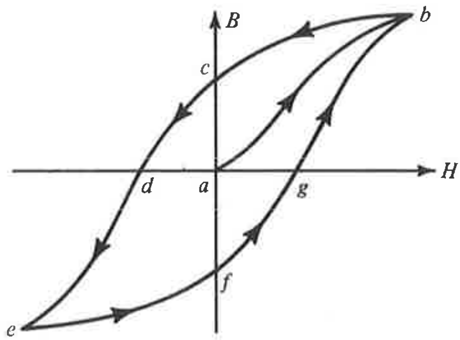

Figure 5.8. Hysteresis curve for a ferromagnetic material.

the manner in which magnetization is built up initially to saturation is given by the portion *ab* of the curve. If the magnetization is now decreased gradually and then reversed in polarity, the curve does not retrace *ab* backward but instead follows along *bcd* until saturation is reached in the opposite direction at point *e*. A decrease in the magnetization back to zero followed by a reversal back to the original polarity brings the point back to *b* along the curve through the points *f* and *g*, thereby completing the loop. A continuous repetition of the process thereafter would simply make the point trace the hysteresis loop *bcdefgb* repeatedly.

5.4 WAVE EQUATION AND SOLUTION

In the previous three sections we introduced conductors, dielectrics, and magnetic materials. We found that conductors are characterized by conduction current, dielectrics are characterized by polarization current, and magnetic materials are characterized by magnetization current. The conduction current density is related to the electric field intensity through the conductivity σ of the conductor. To take into account the effects of polarization, we modified the relationship between **D** and **E** by introducing the permittivity ϵ of the dielectric. Similarly, to take into account the effects of magnetization, we modified the relationship between **H** and **B** by introducing the permeability μ of the magnetic material. The three pertinent relations, known as the "constitutive relations," are

$$\mathbf{J}_c = \sigma \mathbf{E} \tag{5.37a}$$

$$\mathbf{D} = \epsilon \mathbf{E} \tag{5.37b}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} \tag{5.37c}$$

A given material may possess all three properties although usually one of them is predominant. Hence in this section we shall consider a material medium characterized by σ, ϵ, and μ. The Maxwell's curl equations for such a medium are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{5.38}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t} \tag{5.39}$$

To discuss electromagnetic wave propagation in the material medium, let us consider the infinite plane current sheet of Fig. 4.8, except that now the medium on either side of the sheet is a material instead of free space, as shown in Fig. 5.9.

The electric and magnetic fields for the simple case of the infinite plane

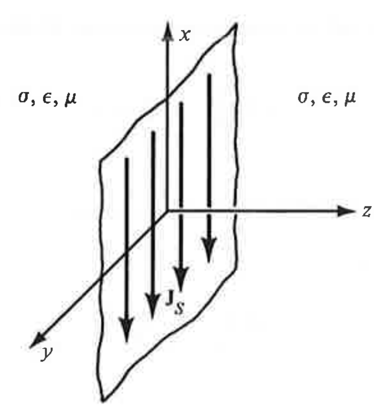

Figure 5.9. Infinite plane current sheet imbedded in a material medium.

current sheet in the $z = 0$ plane and carrying uniformly distributed current in the negative x direction as given by

$$\mathbf{J}_S = -J_{S0} \cos \omega t \, \mathbf{i}_x \tag{5.40}$$

are of the form

$$\mathbf{E} = E_x(z, t)\mathbf{i}_x \tag{5.41a}$$

$$\mathbf{H} = H_y(z, t)\mathbf{i}_y \tag{5.41b}$$

The corresponding simplified forms of the Maxwell's curl equations are

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \tag{5.42}$$

$$\frac{\partial H_y}{\partial z} = -\sigma E_x - \epsilon \frac{\partial E_x}{\partial t} \tag{5.43}$$

We shall make use of the phasor technique to solve these equations. Thus letting

$$E_x(z, t) = \text{Re}\,[\bar{E}_x(z)\, e^{j\omega t}] \tag{5.44a}$$

$$H_y(z, t) = \text{Re}\,[\bar{H}_y(z)\, e^{j\omega t}] \tag{5.44b}$$

and replacing E_x and H_y in (5.42) and (5.43) by their phasors $\bar{E}_x$ and $\bar{H}_y$, respectively, and $\partial/\partial t$ by $j\omega$, we obtain the corresponding differential equations for the phasors $\bar{E}_x$ and $\bar{H}_y$ as

$$\frac{\partial \bar{E}_x}{\partial z} = -j\omega\mu\bar{H}_y \tag{5.45}$$

$$\frac{\partial \bar{H}_y}{\partial z} = -\sigma\bar{E}_x - j\omega\epsilon\bar{E}_x = -(\sigma + j\omega\epsilon)\bar{E}_x \tag{5.46}$$

Differentiating (5.45) with respect to z and using (5.46), we obtain

$$\frac{\partial^2 \bar{E}_x}{\partial z^2} = -j\omega\mu \frac{\partial \bar{H}_y}{\partial z} = j\omega\mu(\sigma + j\omega\epsilon)\bar{E}_x \tag{5.47}$$

Defining

$$\bar{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \tag{5.48}$$

and substituting in (5.47), we have

$$\frac{\partial^2 \bar{E}_x}{\partial z^2} = \bar{\gamma}^2 \bar{E}_x \tag{5.49}$$

Equation (5.49) is the wave equation for $\bar{E}_x$ in the material medium and its solution is given by

$$\bar{E}_x(z) = \bar{A}e^{-\bar{\gamma}z} + \bar{B}e^{\bar{\gamma}z} \tag{5.50}$$

where $\bar{A}$ and $\bar{B}$ are arbitrary constants. Noting that $\bar{\gamma}$ is a complex number and hence can be written as

$$\bar{\gamma} = \alpha + j\beta \tag{5.51}$$

and also writing $\bar{A}$ and $\bar{B}$ in exponential form as $Ae^{j\theta}$ and $Be^{j\phi}$, respectively, we have

$$\bar{E}_x(z) = Ae^{j\theta}e^{-\alpha z}e^{-j\beta z} + Be^{j\phi}e^{\alpha z}e^{j\beta z}$$

or

$$\begin{aligned} E_x(z, t) &= \text{Re}\,[\bar{E}_x(z)\, e^{j\omega t}] \\ &= \text{Re}\,[Ae^{j\theta}e^{-\alpha z}e^{-j\beta z}e^{j\omega t} + Be^{j\phi}e^{\alpha z}e^{j\beta z}e^{j\omega t}] \\ &= Ae^{-\alpha z}\cos(\omega t - \beta z + \theta) + Be^{\alpha z}\cos(\omega t + \beta z + \phi) \end{aligned} \tag{5.52}$$

We now recognize the two terms on the right side of (5.52) as representing uniform plane waves propagating in the positive z and negative z directions, respectively, with phase constant β, in view of the factors $\cos(\omega t - \beta z + \theta)$ and $\cos(\omega t + \beta z + \phi)$, respectively. They are, however, multiplied by the factors $e^{-\alpha z}$ and $e^{\alpha z}$, respectively. Hence the peak amplitude of the field differs from one constant phase surface to another. Since there cannot be a positive going wave in the region $z < 0$, that is, to the left of the current sheet, and since there cannot be a negative going wave in the region $z > 0$, that is, to the right of the current sheet, the solution for the electric field is given by

$$E_x(z, t) = \begin{cases} Ae^{-\alpha z}\cos(\omega t - \beta z + \theta) & \text{for } z > 0 \\ Be^{\alpha z}\cos(\omega t + \beta z + \phi) & \text{for } z < 0 \end{cases} \tag{5.53}$$

To discuss how the peak amplitude of E_x varies with z on either side of the current sheet, we note that since σ, ϵ, and μ are all positive, the phase angle of $j\omega\mu(\sigma + j\omega\epsilon)$ lies between 90° and 180° and hence the phase angle of $\bar{\gamma}$ lies between 45° and 90°, making α and β positive quantities. This means that $e^{-\alpha z}$ decreases with increasing value of z, that is, in the positive z direction, and $e^{\alpha z}$ decreases with decreasing value of z, that is, in the negative z direction. Thus the exponential factors $e^{-\alpha z}$ and $e^{\alpha z}$ associated with the solutions for E_x in (5.53) have the effect of reducing the amplitude of the field, that is, attenuating it as it propagates away from the sheet to either side of it. For this reason, the quantity α is known as the "attenuation constant." The attenuation per unit length is equal to e^{α}. In terms of decibels, this is equal to $20 \log_{10} e^{\alpha}$ or 8.686α db. The units of α are nepers per meter. The quantity $\bar{\gamma}$ is known as the "propagation constant" since its real and imaginary parts, α and β, together determine the propagation characteristics, that is, attenuation and phase shift of the wave.

Returning now to the expression for $\bar{\gamma}$ given by (5.48), we can obtain the expressions for α and β by squaring it on both sides and equating the real and imaginary parts on both sides. Thus

$$\bar{\gamma}^2 = (\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

or

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \tag{5.54a}$$

$$2\alpha\beta = \omega\mu\sigma \tag{5.54b}$$

Now, squaring (5.54a) and (5.54b) and adding and then taking the square root, we obtain

$$\alpha^2 + \beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} \tag{5.55}$$

From (5.54a) and (5.55), we then have

$$\alpha^2 = \frac{1}{2}\left[-\omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\beta^2 = \frac{1}{2}\left[\omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

Since α and β are both positive, we finally get

$$\alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \tag{5.56}$$

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \tag{5.57}$$

We note from (5.56) and (5.57) that α and β are both dependent on σ through the factor $\sigma/\omega\epsilon$. This factor, known as the "loss tangent," is the ratio of the magnitude of the conduction current density $\sigma\bar{E}_x$ to the magnitude of the displacement current density $j\omega\epsilon\bar{E}_x$ in the material medium. In practice, the loss tangent is, however, not simply inversely proportional to ω since both σ and ϵ are generally functions of frequency.

The phase velocity of the wave along the direction of propagation is given by

$$v_p = \frac{\omega}{\beta} = \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2} \tag{5.58}$$

We note that the phase velocity is dependent on the frequency of the wave. Thus waves of different frequencies travel with different phase velocities, that is, they undergo different rates of change of phase with z at any fixed time.

This characteristic of the material medium gives rise to a phenomenon known as "dispersion." We shall discuss dispersion in Chap. 7. The wavelength in the medium is given by

$$\lambda = \frac{2\pi}{\beta} = \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2} \tag{5.59}$$

Having found the solution for the electric field of the wave and discussed its general properties, we now turn to the solution for the corresponding magnetic field by substituting for E_x in (5.45). Thus

$$\begin{aligned}\bar{H}_y &= -\frac{1}{j\omega\mu}\frac{\partial \bar{E}_x}{\partial z} = \frac{\bar{\gamma}}{j\omega\mu}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z}) \\ &= \sqrt{\frac{\sigma + j\omega\epsilon}{j\omega\mu}}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z}) \\ &= \frac{1}{\bar{\eta}}(\bar{A}e^{-\bar{\gamma}z} - \bar{B}e^{\bar{\gamma}z})\end{aligned} \tag{5.60}$$

where

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \tag{5.61}$$

is the intrinsic impedance of the medium. Writing

$$\bar{\eta} = |\bar{\eta}|\,e^{j\tau} \tag{5.62}$$

we obtain the solution for $H_y(z, t)$ as

$$\begin{aligned}H_y(z, t) &= \text{Re}\,[\bar{H}_y(z)\,e^{j\omega t}] \\ &= \text{Re}\left[\frac{1}{|\bar{\eta}|\,e^{j\tau}}Ae^{j\theta}e^{-\alpha z}e^{-j\beta z}e^{j\omega t} - \frac{1}{|\bar{\eta}|\,e^{j\tau}}Be^{j\phi}e^{\alpha z}e^{j\beta z}e^{j\omega t}\right] \\ &= \frac{A}{|\bar{\eta}|}e^{-\alpha z}\cos(\omega t - \beta z + \theta - \tau) - \frac{B}{|\bar{\eta}|}e^{\alpha z}\cos(\omega t + \beta z + \phi - \tau)\end{aligned} \tag{5.63}$$

Remembering that the first and second terms on the right side of (5.63) correspond to (+) and (−) waves, respectively, and hence represent the solutions for the magnetic field in the regions $z > 0$ and $z < 0$, respectively, and recalling that the solution for H_y adjacent to the current sheet is given by

$$H_y = \begin{cases} \dfrac{J_{S0}}{2}\cos\omega t & \text{for } z = 0+ \\ -\dfrac{J_{S0}}{2}\cos\omega t & \text{for } z = 0- \end{cases} \tag{5.64}$$

we obtain

$$A = \frac{|\bar{\eta}| J_{S0}}{2}, \qquad \theta = \tau \tag{5.65a}$$

$$B = \frac{|\bar{\eta}| J_{S0}}{2}, \qquad \phi = \tau \tag{5.65b}$$

Thus the electromagnetic field due to the infinite plane current sheet in the xy plane having

$$\mathbf{J}_S = -J_{S0} \cos \omega t \, \mathbf{i}_x$$

and with a material medium characterized by σ, ϵ, and μ on either side of it is given by

$$\mathbf{E}(z, t) = \frac{|\bar{\eta}| J_{S0}}{2} e^{\mp \alpha z} \cos (\omega t \mp \beta z + \tau) \, \mathbf{i}_x \qquad \text{for } z \gtrless 0 \tag{5.66a}$$

$$\mathbf{H}(z, t) = \pm \frac{J_{S0}}{2} e^{\mp \alpha z} \cos (\omega t \mp \beta z) \, \mathbf{i}_y \qquad \text{for } z \gtrless 0 \tag{5.66b}$$

We note from (5.66a) and (5.66b) that wave propagation in the material medium is characterized by phase difference between **E** and **H** in addition to attenuation. These properties are illustrated in Fig. 5.10, which shows sketches of the current density on the sheet and the distance–variation of the electric and magnetic fields on either side of the current sheet for a few values of t.

Since the fields are attenuated as they progress in their respective directions of propagation, the medium is characterized by power dissipation. In fact, by evaluating the power flow out of a rectangular box lying between z and $z + \Delta z$ and having dimensions Δx and Δy in the x and y directions, respectively, as was done in Sect. 4.6, we obtain

$$\begin{aligned}
\oint_S \mathbf{P} \cdot d\mathbf{S} &= \frac{\partial P_z}{\partial z} \Delta x \, \Delta y \, \Delta z = \frac{\partial}{\partial z}(E_x H_y) \, \Delta v \\
&= \left(E_x \frac{\partial H_y}{\partial z} + H_y \frac{\partial E_x}{\partial z} \right) \Delta v \\
&= \left[E_x \left(-\sigma E_x - \epsilon \frac{\partial E_x}{\partial t} \right) + H_y \left(-\mu \frac{\partial H_y}{\partial t} \right) \right] \Delta v \\
&= -\sigma E_x^2 \, \Delta v - \frac{\partial}{\partial t}\left(\frac{1}{2} \epsilon E_x^2 \, \Delta v \right) - \frac{\partial}{\partial t}\left(\frac{1}{2} \mu H_y^2 \, \Delta v \right)
\end{aligned} \tag{5.67}$$

The quantity $\sigma E_x^2 \, \Delta v$ is obviously the power dissipated in the volume Δv due to attenuation and the quantities $\frac{1}{2} \epsilon E_x^2 \, \Delta v$ and $\frac{1}{2} \mu H_y^2 \, \Delta v$ are the energies

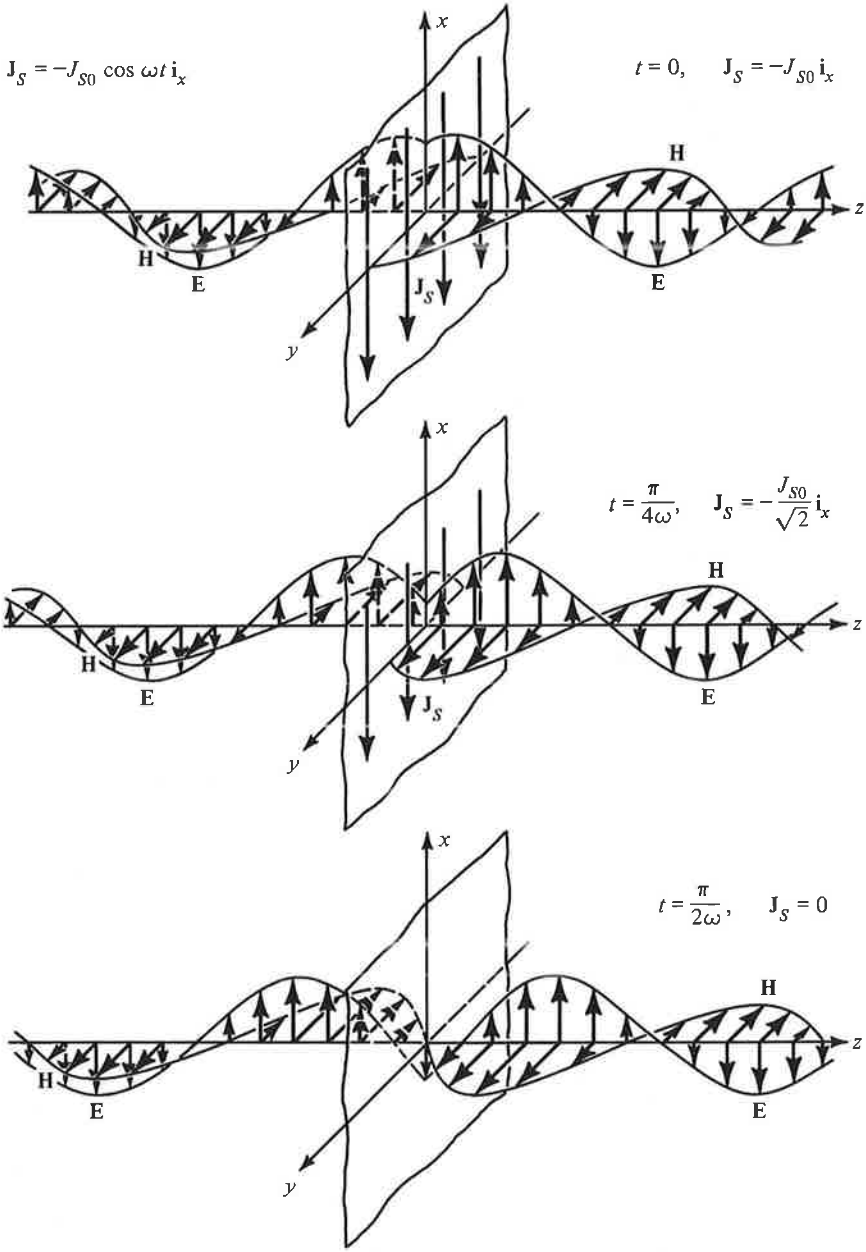

Figure 5.10. Time history of uniform plane electromagnetic wave radiating away from an infinite plane current sheet imbedded in a material medium.

stored in the electric and magnetic fields, respectively, in the volume Δv. It then follows that the power dissipation density, the stored energy density associated with the electric field and the stored energy density associated with the magnetic field are given by

$$P_d = \sigma E_x^2 \tag{5.68}$$

$$w_e = \frac{1}{2}\epsilon E_x^2 \tag{5.69}$$

and

$$w_m = \frac{1}{2}\mu H_y^2 \tag{5.70}$$

respectively. Equation (5.67) is the generalization, to the material medium, of the Poynting's theorem given by (4.71) for free space.

5.5 UNIFORM PLANE WAVES IN DIELECTRICS

In the previous section we discussed electromagnetic wave propagation for the general case of a material medium characterized by conductivity σ, permittivity ϵ, and permeability μ. We found general expressions for the attenuation constant α, the phase constant β, the phase velocity v_p, the wavelength λ, and the intrinsic impedance $\bar{\eta}$. These are given by (5.56), (5.57), (5.58), (5.59), and (5.61), respectively. For $\sigma = 0$, the medium is a "perfect dielectric," having the propagation characteristics

$$\alpha = 0 \tag{5.71a}$$

$$\beta = \omega\sqrt{\mu\epsilon} \tag{5.71b}$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \tag{5.71c}$$

$$\lambda = \frac{1}{f\sqrt{\mu\epsilon}} \tag{5.71d}$$

$$\bar{\eta} = \sqrt{\frac{\mu}{\epsilon}} \tag{5.71e}$$

Thus the waves propagate without attenuation as in free space but with ϵ_0 and μ_0 replaced by ϵ and μ, respectively. For nonzero σ, there are two special cases: (a) imperfect dielectrics or poor conductors and (b) good conductors. The first case is characterized by conduction current small in magnitude compared to the displacement current; the second case is characterized by just the opposite. We shall consider the first case in this section and the second case in the following section.

Thus considering the case of "imperfect dielectrics," we have $|\sigma\bar{E}_x| \ll |j\omega\epsilon\bar{E}_x|$, or $\sigma/\omega\epsilon \ll 1$. We can then obtain approximate expressions for α, β, v_p, λ, and $\bar{\eta}$ as follows:

$$\begin{aligned}
\alpha &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} \\
&= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[1+\frac{\sigma^2}{2\omega^2\epsilon^2}-\frac{\sigma^4}{8\omega^4\epsilon^4}+\dots-1\right]^{1/2} \\
&\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\,\frac{\sigma}{\sqrt{2}\,\omega\epsilon}\left[1-\frac{\sigma^2}{4\omega^2\epsilon^2}\right]^{1/2} \\
&\approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72a}$$

$$\begin{aligned}
\beta &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2} \\
&\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[2+\frac{\sigma^2}{2\omega^2\epsilon^2}\right]^{1/2} \\
&\approx \omega\sqrt{\mu\epsilon}\left(1+\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72b}$$

$$\begin{aligned}
v_p &= \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[2+\frac{\sigma^2}{2\omega^2\epsilon^2}\right]^{-1/2} \\
&\approx \frac{1}{\sqrt{\mu\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72c}$$

$$\begin{aligned}
\lambda &= \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{-1/2} \\
&\approx \frac{1}{f\sqrt{\mu\epsilon}}\left(1-\frac{\sigma^2}{8\omega^2\epsilon^2}\right)
\end{aligned} \tag{5.72d}$$

$$\begin{aligned}
\bar{\eta} &= \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}}\left(1-j\frac{\sigma}{\omega\epsilon}\right)^{-1/2} \\
&= \sqrt{\frac{\mu}{\epsilon}}\left[1+j\frac{\sigma}{2\omega\epsilon}-\frac{3}{8}\frac{\sigma^2}{\omega^2\epsilon^2}-\dots\right] \\
&\approx \sqrt{\frac{\mu}{\epsilon}}\left[\left(1-\frac{3}{8}\frac{\sigma^2}{\omega^2\epsilon^2}\right)+j\frac{\sigma}{2\omega\epsilon}\right]
\end{aligned} \tag{5.72e}$$

In (5.72a)–(5.72e) we have retained all terms up to and including the second power in $\sigma/\omega\epsilon$ and have neglected all higher-order terms. For a value of $\sigma/\omega\epsilon$ equal to 0.1, the quantities β, v_p, and λ are different from those for the corresponding perfect dielectric case by a factor of only 0.01/8 or $\frac{1}{800}$ whereas the intrinsic impedance has a real part differing from the intrinsic impedance of the perfect dielectric medium by a factor of $\frac{3}{800}$ and an imaginary part which is $\frac{1}{20}$ of the intrinsic impedance of the perfect dielectric medium. Thus the only significant feature different from the perfect dielectric case is the attenuation.

Example 5.2. Let us consider that a material can be classified as a dielectric for $\sigma/\omega\epsilon < 0.1$ and compute the values of the several propagation parameters for three materials: mica, dry earth, and sea water.

Denoting the frequency for which $\sigma/\omega\epsilon = 1$ as f_q, we have $f_q = \sigma/2\pi\epsilon$, assuming that σ and ϵ are independent of frequency. Values of σ, ϵ, and f_q and approximate values of the several propagation parameters for $f > 10f_q$ are listed in Table 5.3, in which c is the velocity of light in free space and β_0

TABLE 5.3. Values of Several Propagation Parameters for Three Materials for the Dielectric Range of Frequencies

Material	σ ℧/m	ϵ_r	f_q Hz	α Np/m	β/β_0	v_p/c	λ/λ_0	$\bar{\eta}$ ohms
Mica	10^{-11}	6	3×10^{-2}	77×10^{-11}	2.45	0.408	0.408	153.9
Dry earth	10^{-5}	5	3.6×10^{4}	84×10^{-5}	2.24	0.447	0.447	168.6
Sea water	4	80	0.9×10^{9}	84.3	8.94	0.112	0.112	42.15

and λ_0 are the phase constant and wavelength in free space for the frequency of operation. It can be seen from Table 5.3 that mica behaves as a dielectric for almost any frequency, but sea water can be classified as a dielectric only for frequencies above approximately 10 GHz. We also note that because of the low value of α, mica is a good dielectric, but the high value of α for sea water makes it a poor dielectric. ■

5.6 UNIFORM PLANE WAVES IN CONDUCTORS

In the previous section we considered the special case of imperfect dielectrics. Turning now to the case of "good conductors," we have $|\sigma\bar{E}_x| \gg |j\omega\epsilon\bar{E}_x|$, or $\sigma/\omega\epsilon \gg 1$. We can then obtain approximate expressions for α, β, v_p, λ, and η as follows:

$$\begin{aligned}\alpha &= \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} \\ &\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}} \\ &= \sqrt{\pi f\mu\sigma} \end{aligned} \tag{5.73a}$$

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}$$
$$\approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}}$$
$$= \sqrt{\pi f \mu\sigma} \tag{5.73b}$$
$$v_p = \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2}$$
$$\approx \frac{\sqrt{2}}{\sqrt{\mu\epsilon}}\sqrt{\frac{\omega\epsilon}{\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$
$$= \sqrt{\frac{4\pi f}{\mu\sigma}} \tag{5.73c}$$
$$\lambda = \frac{\sqrt{2}}{f\sqrt{\mu\epsilon}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{-1/2}$$
$$\approx \sqrt{\frac{4\pi}{f\mu\sigma}} \tag{5.73d}$$
$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}}$$
$$= (1 + j)\sqrt{\frac{\pi f \mu}{\sigma}} \tag{5.73e}$$

We note that α, β, v_p, and $\bar{\eta}$ are proportional to $\sqrt{f}$, provided that σ and μ are constants.

To discuss the propagation characteristics of a wave inside a good conductor, let us consider the case of copper. The constants for copper are $\sigma = 5.80 \times 10^7$ mho/m, $\epsilon = \epsilon_0$, and $\mu = \mu_0$. Hence the frequency at which σ is equal to $\omega\epsilon$ for copper is equal to $5.8 \times 10^7/2\pi\epsilon_0$ or 1.04×10^{18} Hz. Thus at frequencies of even several gigahertz, copper behaves like an excellent conductor. To obtain an idea of the attenuation of the wave inside the conductor, we note that the attenuation undergone in a distance of one wavelength is equal to $e^{-\alpha\lambda}$ or $e^{-2\pi}$. In terms of decibels, this is equal to $20 \log_{10} e^{2\pi} = 54.58$ db. In fact, the field is attenuated by a factor e^{-1} or 0.368 in a distance equal to $1/\alpha$. This distance is known as the "skin depth" and is denoted by the symbol δ. From (5.73a), we obtain

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} \tag{5.74}$$

The skin depth for copper is equal to

$$\frac{1}{\sqrt{\pi f \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}} = \frac{0.066}{\sqrt{f}} \text{ m.}$$

Thus in copper the fields are attenuated by a factor e^{-1} in a distance of 0.066 mm even at the low frequency of 1 MHz, thereby resulting in the concentration of the fields near to the skin of the conductor. This phenomenon is known as the "skin effect." It also explains "shielding" by conductors.

To discuss further the characteristics of wave propagation in a good conductor, we note that the ratio of the wavelength in the conducting medium to the wavelength in a dielectric medium having the same ϵ and μ as those of the conductor is given by

$$\frac{\lambda_{\text{conductor}}}{\lambda_{\text{dielectric}}} \approx \frac{\sqrt{4\pi/f\mu\sigma}}{1/f\sqrt{\mu\epsilon}} = \sqrt{\frac{4\pi f\epsilon}{\sigma}} = \sqrt{\frac{2\omega\epsilon}{\sigma}} \tag{5.75}$$

Since $\sigma/\omega\epsilon \gg 1$, $\lambda_{\text{conductor}} \ll \lambda_{\text{dielectric}}$. For example, for sea water, $\sigma = 4$ mhos/m, $\epsilon = 80\epsilon_0$, and $\mu = \mu_0$ so that the ratio of the two wavelengths for $f = 25$ kHz is equal to 0.00745. Thus for $f = 25$ kHz, the wavelength in sea water is $\frac{1}{134}$ of the wavelength in a dielectric having the same ϵ and μ as those of sea water and a still smaller fraction of the wavelength in free space. Furthermore, the lower the frequency, the smaller is this fraction. Since it is the electrical length, that is, the length in terms of the wavelength, instead of the physical length that determines the radiation efficiency of an antenna, this means that antennas of much shorter length can be used in sea water than in free space. Together with the property that $\alpha \propto \sqrt{f}$, this illustrates that low frequencies are more suitable than high frequencies for communication under water, and with underwater objects.

Equation (5.73e) tells us that the intrinsic impedance of a good conductor has a phase angle of 45°. Hence the electric and magnetic fields in the medium are out of phase by 45°. The magnitude of the intrinsic impedance is given by

$$|\bar{\eta}| = \left|(1+j)\sqrt{\frac{\pi f\mu}{\sigma}}\right| = \sqrt{\frac{2\pi f\mu}{\sigma}} \tag{5.76}$$

As a numerical example, for copper, this quantity is equal to

$$\sqrt{\frac{2\pi f \times 4\pi \times 10^{-7}}{5.8 \times 10^{7}}} = 3.69 \times 10^{-7}\sqrt{f} \text{ ohms}$$

Thus the intrinsic impedance of copper has as low a magnitude as 0.369 ohms even at a frequency of 10^{12} Hz. In fact, by recognizing that

$$|\bar{\eta}| = \sqrt{\frac{2\pi f\mu}{\sigma}} = \sqrt{\frac{\omega\epsilon}{\sigma}}\sqrt{\frac{\mu}{\epsilon}} \tag{5.77}$$

we note that the magnitude of the intrinsic impedance of a good conductor medium is a small fraction of the intrinsic impedance of a dielectric medium having the same ϵ and μ. It follows that for the same electric field, the magnetic field inside a good conductor is much larger than the magnetic field inside a dielectric having the same ϵ and μ as those of the conductor.

Finally, for $\sigma = \infty$, the medium is a "perfect conductor," an idealization of the good conductor. From (5.74), we note that the skin depth is then equal to zero and that there is no penetration of the fields. Thus no fields can exist inside a perfect conductor.

5.7 SUMMARY

In this chapter we studied the principles of uniform plane wave propagation in a material medium. Material media can be classified as (a) conductors, (b) dielectrics, and (c) magnetic materials, depending on the nature of the response of the charged particles in the materials to applied fields. Conductors are characterized by conduction which is the phenomenon of steady drift of free electrons under the influence of an applied electric field. Dielectrics are characterized by polarization which is the phenomenon of the creation and net alignment of electric dipoles, formed by the displacement of the centroids of the electron clouds from the centroids of the nucleii of the atoms, along the direction of an applied electric field. Magnetic materials are characterized by magnetization which is the phenomenon of net alignment of the axes of the magnetic dipoles, formed by the electron orbital and spin motion around the nucleii of the atoms, along the direction of an applied magnetic field.

Under the influence of applied electromagnetic wave fields, all three phenomena described above give rise to currents in the material which in turn influence the wave propagation. These currents are known as the conduction, polarization, and magnetization currents, respectively, for conductors, dielectrics, and magnetic materials. They must be taken into account in the first term on the right side of Ampere's circuital law, that is, $\int_S \mathbf{J} \cdot d\mathbf{S}$ in the case of the integral form and $\mathbf{J}$ in the case of the differential form. The conduction current density is given by

$$\mathbf{J}_c = \sigma \mathbf{E} \tag{5.78}$$

where σ is the conductivity of the material. The conduction current is taken into account explicitly by replacing $\mathbf{J}$ by $\mathbf{J}_c$. The polarization and magnetization currents are taken into account implicitly by revising the definitions of the displacement flux density vector and the magnetic field intensity vector to read as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{5.79}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{5.80}$$

where **P** and **M** are the polarization and magnetization vectors, respectively. For linear isotropic materials, (5.79) and (5.80) simplify to

$$\mathbf{D} = \epsilon \mathbf{E} \tag{5.81}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} \tag{5.82}$$

where

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\mu = \mu_0 \mu_r$$

are the permittivity and the permeability, respectively, of the material. The quantities ϵ_r and μ_r are the relative permittivity and the relative permeability, respectively, of the material. The parameters σ, ϵ, and μ vary from one material to another and are in general dependent on the frequency of the wave. Equations (5.78), (5.81), and (5.82) are known as the constitutive relations. For anisotropic materials, these relations are expressed in the form of matrix equations with the material parameters represented by tensors.

Together with Maxwell's equations, the constitutive relations govern the behavior of the electromagnetic field in a material medium. Thus Maxwell's curl equations for a material medium are given by

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

We made use of these equations for the simple case of $\mathbf{E} = E_x(z, t)\mathbf{i}_x$ and $\mathbf{H} = H_y(z, t)\mathbf{i}_y$ to obtain the uniform plane wave solution by considering the infinite plane current sheet in the xy plane with uniform surface current density

$$\mathbf{J}_S = -\mathbf{J}_{S0} \cos \omega t \, \mathbf{i}_x$$

and with a material medium on either side of it and finding the electromagnetic field due to the current sheet to be given by

$$\mathbf{E} = \frac{|\bar{\eta}| J_{S0}}{2} e^{\mp \alpha z} \cos (\omega t \mp \beta z + \tau) \, \mathbf{i}_x \qquad \text{for } z \gtrless 0 \tag{5.83a}$$

$$\mathbf{H} = \pm \frac{J_{S0}}{2} e^{\mp \alpha z} \cos (\omega t \mp \beta z) \, \mathbf{i}_y \qquad \text{for } z \gtrless 0 \tag{5.83b}$$

In (5.83a–b), α and β are the attenuation and phase constants given, respectively, by the real and imaginary parts of the propagation constant, $\bar{\gamma}$. Thus

$$\bar{\gamma} = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

The quantities $|\bar{\eta}|$ and τ are the magnitude and phase angle, respectively, of the intrinsic impedance, $\bar{\eta}$, of the medium. Thus

$$\bar{\eta} = |\bar{\eta}|\,e^{j\tau} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

The uniform plane wave solution given by (5.83a–b) tells us that the wave propagation in the material medium is characterized by attenuation as indicated by $e^{\mp\alpha z}$ and phase difference between **E** and **H** by the amount τ. We learned that the attenuation of the wave results from power dissipation due to conduction current flow in the medium. The power dissipation density is given by

$$p_d = \sigma E_x^2$$

The stored energy densities associated with the electric and magnetic fields in the medium are given by

$$w_e = \frac{1}{2}\epsilon E^2$$

$$w_m = \frac{1}{2}\mu H^2$$

Having discussed uniform plane wave propagation for the general case of a medium characterized by σ, ϵ, and μ, we then considered several special cases. These are discussed in the following:

PERFECT DIELECTRICS: For these materials, $\sigma = 0$. Wave propagation occurs without attenuation as in free space but with the propagation parameters governed by ϵ and μ instead of ϵ_0 and μ_0, respectively.

IMPERFECT DIELECTRICS: A material is classified as an imperfect dielectric for $\sigma \ll \omega\epsilon$, that is, conduction current density is small in magnitude compared to the displacement current density. The only significant feature of wave propagation in an imperfect dielectric as compared to that in a perfect dielectric is the attenuation undergone by the wave.

GOOD CONDUCTORS: A material is classified as a good conductor for $\sigma \gg \omega\epsilon$, that is, conduction current density is large in magnitude compared to the displacement current density. Wave propagation in a good conductor medium is characterized by attenuation and phase constants both equal to $\sqrt{\pi f \mu\sigma}$. Thus for large values of f and/or σ, the fields do not penetrate very deeply into the conductor. This phenomenon is known as the skin effect. From considerations of the frequency dependence of the attenuation and wavelength for a fixed σ, we learned that low frequencies are more suitable for communication with underwater objects. We also learned that the intrinsic impedance of a good conductor medium is very low in magnitude compared to that of a dielectric medium having the same ϵ and μ.

PERFECT CONDUCTORS: These are idealizations of good conductors in the limit $\sigma \longrightarrow \infty$. For $\sigma = \infty$, the skin depth, that is, the distance in which the fields inside a conductor are attenuated by a factor e^{-1}, is zero and hence there can be no penetration of fields into a perfect conductor.

REVIEW QUESTIONS

5.1. Distinguish between bound electrons and free electrons in an atom.

5.2. Briefly describe the phenomenon of conduction.

5.3. State Ohms' law applicable at a point. How is it taken into account in Maxwell's equations?

5.4. Briefly describe the phenomenon of polarization in a dielectric material.

5.5. What is an electric dipole? How is its strength defined?

5.6. What are the different kinds of polarization in a dielectric?

5.7. What is the polarization vector? How is it related to the electric field intensity?

5.8. Discuss how polarization current arises in a dielectric material.

5.9. State the relationship between polarization current density and electric field intensity. How is it taken into account in Maxwell's equations?

5.10. What is the revised definition of **D**?

5.11. State the relationship between **D** and **E** in a dielectric material. How does it simplify the solution of field problems involving dielectrics?

5.12. What is an anisotropic dielectric material?

5.13. When can an effective permittivity be defined for an anisotropic dielectric material?

5.14. Briefly describe the phenomenon of magnetization.

5.15. What is a magnetic dipole? How is its strength defined?

5.16. What are the different kinds of magnetic materials?

5.17. What is the magnetization vector? How is it related to the magnetic flux density?

5.18. Discuss how magnetization current arises in a magnetic material.

5.19. State the relationship between magnetization current density and magnetic flux density. How is it taken into account in Maxwell's equations?

5.20. What is the revised definition of **H**?

5.21. State the relationship between **H** and **B** for a magnetic material. How does it simplify the solution of field problems involving magnetic materials?

5.22. What is an anisotropic magnetic material?

5.23. Discuss the relationship between B and H for a ferromagnetic material.

5.24. Summarize the constitutive relations for a material medium.

5.25. What is the propagation constant for a material medium? Discuss the significance of its real and imaginary parts.

5.26. Discuss the consequence of the frequency dependence of the phase velocity of a wave in a material medium.

5.27. What is loss tangent? Discuss its significance.

5.28. What is the intrinsic impedance of a material medium? What is the consequence of its complex nature?

5.29. How do you account for the attenuation undergone by the wave in a material medium?

5.30. What is the power dissipation density in a medium characterized by nonzero conductivity?

5.31. What are the stored energy densities associated with electric and magnetic fields in a material medium?

5.32. What is the condition for a medium to be a perfect dielectric? How do the characteristics of wave propagation in a perfect dielectric medium differ from those of wave propagation in free space?

5.33. What is the criterion for a material to be an imperfect dielectric? What is the significant feature of wave propagation in an imperfect dielectric as compared to that in a perfect dielectric?

5.34. Give two examples of materials that behave as good dielectrics for frequencies down to almost zero.

5.35. What is the criterion for a material to be a good conductor?

5.36. Give two examples of materials that behave as good conductors for frequencies of up to several gigahertz.

5.37. What is skin effect? Discuss skin depth, giving some numerical values.

5.38. Why are low-frequency waves more suitable than high-frequency waves for communication with underwater objects?

5.39. Discuss the consequence of the low intrinsic impedance of a good conductor as compared to that of a dielectric medium having the same ϵ and μ.

5.40. Why can there be no fields inside a perfect conductor?

PROBLEMS

5.1. Find the electric field intensity required to produce a current of 0.1 amp crossing an area of 1 cm^2 normal to the field for the following materials: (a) copper, (b) aluminum, and (c) sea water. Then find the voltage drop along a length of 1 cm parallel to the field, and find the ratio of the voltage drop to the current (resistance) for each material.

5.2. The free electron density in silver is 5.80×10^{28} m^{-3}. (a) Find the mobility of the electron for silver. (b) Find the drift velocity of the electrons for an applied electric field of intensity 0.1 V/m.

5.3. Use the continuity equation, Ohm's law, and Gauss' law for the electric field to show that the time variation of the charge density at a point inside a conductor is governed by the differential equation

$$\frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon_0}\rho = 0$$

Then show that the charge density inside the conductor decays exponentially with a time constant ϵ_0/σ. Compute the value of the time constant for copper.

5.4. Show that the torque acting on an electric dipole of moment $\mathbf{p}$ due to an applied electric field $\mathbf{E}$ is $\mathbf{p} \times \mathbf{E}$.

5.5. For an applied electric field $\mathbf{E} = 0.1 \cos 2\pi \times 10^9 t\, \mathbf{i}_x$ V/m, find the polarization current crossing an area of 1 cm^2 normal to the field for the following materials: (a) polystyrene, (b) mica, and (c) distilled water.

5.6. For the anisotropic dielectric material having the permittivity tensor given in Example 5.1, find $\mathbf{D}$ for $\mathbf{E} = E_0 (\cos \omega t\, \mathbf{i}_x + \sin \omega t\, \mathbf{i}_y)$. Comment on your result.

5.7. An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \epsilon_0 \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

(a) Find $\mathbf{D}$ for $\mathbf{E} = E_0\mathbf{i}_x$. (b) Find $\mathbf{D}$ for $\mathbf{E} = E_0(\mathbf{i}_x + \mathbf{i}_y + \mathbf{i}_z)$. (c) Find $\mathbf{E}$ which produces $\mathbf{D} = 4\epsilon_0 E_0 \mathbf{i}_x$.

5.8. An anisotropic dielectric material is characterized by the permittivity tensor

$$[\epsilon] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

For $\mathbf{E} = (E_x\mathbf{i}_x + E_y\mathbf{i}_y) \cos \omega t$, find the value(s) of E_y/E_x for which $\mathbf{D}$ is parallel to $\mathbf{E}$. Find the effective permittivity for each case.

5.9. Find the magnetic dipole moment of an electron in circular orbit of radius a normal to a uniform magnetic field of flux density B_0. Compute its value for $a = 10^{-3}$ m and $B_0 = 5 \times 10^{-5}$ Wb/m^2.

5.10. Show that the torque acting on a magnetic dipole of moment $\mathbf{m}$ due to an applied magnetic field $\mathbf{B}$ is $\mathbf{m} \times \mathbf{B}$. For simplicity, consider a rectangular loop in the xy plane and $\mathbf{B} = B_x\mathbf{i}_x + B_y\mathbf{i}_y + B_z\mathbf{i}_z$.

5.11. For an applied magnetic field $\mathbf{B} = 10^{-6} \cos 2\pi z\, \mathbf{i}_y$ Wb/m^2, find the magnetization current crossing an area 1 cm^2 normal to the x direction for a magnetic material having $\chi_m = 10^{-3}$.

5.12. An anisotropic magnetic material is characterized by the permeability tensor

$$[\mu] = \mu_0 \begin{bmatrix} 7 & 6 & 0 \\ 6 & 12 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find the effective permeability for $\mathbf{H} = H_0(3\mathbf{i}_x - 2\mathbf{i}_y) \cos \omega t$.

5.13. Obtain the wave equation for $\bar{H}_y$ similar to that for $\bar{E}_x$ given by (5.49).

5.14. Obtain the expression for the attenuation per wavelength undergone by a uniform plane wave in a material medium characterized by σ, ϵ, and μ. Using the logarithmic scale for $\sigma/\omega\epsilon$, plot the attenuation per wavelength in decibels versus $\sigma/\omega\epsilon$.

5.15. For dry earth, $\sigma = 10^{-5}$ mho/m, $\epsilon = 5\epsilon_0$, and $\mu = \mu_0$. Compute α, β, v_p, λ, and $\bar{\eta}$ for $f = 100$ kHz.

5.16. Obtain the expressions for the real and imaginary parts of the intrinsic impedance of a material medium given by (5.61).

5.17. An infinite plane sheet lying in the xy plane carries current of uniform density

$$\mathbf{J}_S = -0.1 \cos 2\pi \times 10^6 t\, \mathbf{i}_x \text{ amp/m}$$

The medium on either side of the sheet is characterized by $\sigma = 10^{-3}$ mho/m, $\epsilon = 18\epsilon_0$, and $\mu = \mu_0$. Find **E** and **H** on either side of the current sheet.

5.18. Repeat Problem 5.17 for

$$\mathbf{J}_S = -0.1(\cos 2\pi \times 10^6 t\, \mathbf{i}_x + \cos 4\pi \times 10^6 t\, \mathbf{i}_x) \text{ amp/m}$$

5.19. For an array of two infinite plane parallel current sheets of uniform densities situated in a medium characterized by $\sigma = 10^{-3}$ mho/m, $\epsilon = 18\epsilon_0$, and $\mu = \mu_0$, find the spacing and the relative amplitudes and phase angles of the current densities to obtain an endfire radiation characteristic for $f = 10^6$ Hz.

5.20. Show that energy is not stored equally in the electric and magnetic fields in a material medium for $\sigma \neq 0$.

5.21. The electric field of a uniform plane wave propagating in a perfect dielectric medium having $\mu = \mu_0$ is given by

$$\mathbf{E} = 10 \cos (6\pi \times 10^7 t - 0.4\pi z)\, \mathbf{i}_x \text{ V/m}$$

Find (a) the frequency, (b) the wavelength, (c) the phase velocity, (d) the permittivity of the medium, and (e) the associated magnetic field vector **H**.

5.22. The electric and magnetic fields of a uniform plane wave propagating in a perfect dielectric medium are given by

$$\mathbf{E} = 10 \cos (6\pi \times 10^7 t - 0.8\pi z)\, \mathbf{i}_x \text{ V/m}$$

$$\mathbf{H} = \frac{1}{6\pi} \cos (6\pi \times 10^7 t - 0.8\pi z)\, \mathbf{i}_y \text{ amp/m}$$

Find the permittivity and the permeability of the medium.

5.23. An infinite plane sheet situated in the xy plane carries a current of uniform density

$$\mathbf{J}_S = -0.2 \cos 3\pi \times 10^7 t\, \mathbf{i}_x \text{ amp/m}$$

The medium on either side of the current sheet is a perfect dielectric having $\epsilon = 8\epsilon_0$ and $\mu = 2\mu_0$. (a) Find **H**, **B**, **M**, and $\mathbf{J}_m$ for $z > 0$. (b) Find **E**, **D**, **P**, and $\mathbf{J}_p$ for $z > 0$.

5.24. Compute f_q for each of the following materials: (a) fused quartz, (b) bakelite, and (c) distilled water. Then compute for the imperfect dielectric range of frequencies the values of α, β, v_p, λ, and $\bar{\eta}$ for each material.

5.25. For uniform plane wave propagation in fresh water ($\sigma = 10^{-3}$ mho/m, $\epsilon = 80\epsilon_0$, $\mu = \mu_0$), find α, β, v_p, λ, and $\bar{\eta}$ for two frequencies: (a) 100 MHz, and (b) 10 kHz.

5.26. Show that for a given material, the ratio of the attenuation constant for the good conductor range of frequencies to the attenuation constant for the imperfect dielectric range of frequencies is equal to $\sqrt{2\omega\epsilon/\sigma}$ where ω is in the good conductor range of frequencies.

5.27. For a 25-kHz wave propagating in sea water, find the Doppler shift observed by an observer, moving with a velocity 10 m/s along the direction of propagation of the wave.