APPENDIX

UNITS AND DIMENSIONS

In 1960, the International System of Units was given official status at the Eleventh General Conference on weights and measures held in Paris, France. This system of units is an expanded version of the rationalized meter-kilogram-second-ampere (MKSA) system of units and is based on six fundamental or basic units. The six basic units are the units of length, mass, time, current, temperature and luminous intensity.

The international unit of length is the meter. It is exactly 1,650,763.73 times the wavelength in vacuum of the radiation corresponding to the unperturbed transition between the levels $2p_{10}$ and $5d_5$ of the atom of krypton-86, the orange-red line. The international unit of mass is the kilogram. It is the mass of the International Prototype Kilogram which is a particular cylinder of platinum-iridium alloy preserved in a vault at Sèvres, France, by the International Bureau of Weights and Measures. The international unit of time is the second. It is equal to 9,192,631,770 times the period corresponding to the frequency of the transition between the hyperfine levels $F = 4$, $M = 0$ and $F = 3$, $M = 0$ of the fundamental state ${}^2S_{1/2}$ of the cesium-133 atom unperturbed by external fields.

To present the definition for the international unit of current, we first define the newton, which is the unit of force, derived from the fundamental units meter, kilogram and second in the following manner. Since velocity is rate of change of distance with time, its unit is meter per second. Since acceleration is rate of change of velocity with time, its unit is meter per second per second or meter per second squared. Since force is mass times acceleration, its unit is kilogram-meter per second squared, also known as the newton. Thus, the newton is that force which imparts an acceleration of 1 meter per second squared to a mass of 1 kilogram. The international unit of current, which is the ampere, can now be defined. It is the constant current which when maintained in two straight, infinitely long, parallel conductors of negligible cross section and placed one meter apart in vacuum produces a force of 2×10^{-7} newtons per meter length of the conductors.

The international unit of temperature is the Kelvin degree. It is based on the definition of the thermodynamic scale of temperature by means of the triple-point of water as a fixed fundamental point to which a temperature of exactly 273.16 degrees Kelvin is attributed. The international unit of luminous intensity is the candela. It is defined such that the luminance of a blackbody radiator at the freezing temperature of platinum is 60 candelas per square centimeter.

We have just defined the six basic units of the International System of Units. Two supplementary units are the radian and the steradian for plane angle and solid angle respectively. All other units are derived units. For example, the unit of charge which is the coulomb is the amount of charge transported in 1 second by a current of 1 ampere; the unit of energy which is joule is the work done when the point of application of a force of 1 newton is displaced a distance of 1 meter in the direction of the force; the unit of power which is the watt is the power which gives rise to the production of energy at the rate of 1 joule per second; the unit of electric potential difference which is the volt is the difference of electric potential between two points of a conducting wire carrying constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt; and so on. The units for the various quantities used in this book are listed in Table A.1., together with the symbols of the quantities and their dimensions.

Dimensions are a convenient means of checking the possible validity of a derived equation. The dimension of a given quantity can be expressed as some combination of a set of fundamental dimensions. These fundamental dimensions need not be the same as the quantities corresponding to the basic units. In mechanics, the fundamental dimensions are mass (M), length (L) and time (T). In electromagnetics, it is the usual practice to consider the charge (Q), instead of the current, as the additional fundamental dimension. For the quantities listed in Table A.1., these four dimensions are sufficient. Thus, for example, the dimension of velocity is length (L) divided by time (T), that is LT^{-1}; the dimension of acceleration is length (L) divided by time squared (T^2), that is, LT^{-2}; the dimension of force is mass (M) times acceleration (LT^{-2}), that is, MLT^{-2}; the dimension of ampere is charge (Q) divided by time (T), that is, QT^{-1}; and so on.

To illustrate the application of dimensions for checking the possible validity of a derived equation, let us consider the equation for the velocity of propagation of an electromagnetic wave in free space, given by

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

TABLE A.1. Symbols, Units and Dimensions of Various Quantities

Quantity	*Symbol*	*Unit*	*Dimensions*
Acceleration	$\mathbf{a}$	meter/(second)^2	LT^{-2}
Admittance	$\bar{Y}$	mho	$M^{-1}L^{-2}TQ^2$
Angular velocity	ω	radian/second	T^{-1}
Area	A	square meter	L^2
Attenuation constant	α	neper/meter	L^{-1}
Capacitance	C	farad	$M^{-1}L^{-2}T^2Q^2$
Capacitance per unit length	$\mathcal{C}$	farad/meter	$M^{-1}L^{-3}T^2Q^2$
Cartesian coordinates	x	meter	L
	y	meter	L
	z	meter	L
Characteristic admittance	Y_0	mho	$M^{-1}L^{-2}TQ^2$
Characteristic impedance	Z_0	ohm	$ML^2T^{-1}Q^{-2}$
Charge	Q, q	coulomb	Q
Closed path	C	meter	L
Conductance	G	mho	$M^{-1}L^{-2}TQ^2$
Conductance per unit length	$\mathcal{G}$	mho/meter	$M^{-1}L^{-3}TQ^2$
Conduction current	I_c	ampere	$T^{-1}Q$
Conduction current density	$\mathbf{J}_c$	ampere/square meter	$L^{-2}T^{-1}Q$
Conductivity	σ	mho/meter	$M^{-1}L^{-3}TQ^2$
Current	I	ampere	$T^{-1}Q$
Current transmission coefficient	τ_c	—	—
Cutoff frequency	f_c	hertz	T^{-1}
Cutoff wavelength	λ_c	meter	L
Cylindrical coordinates	r, r_c	meter	L
	ϕ	radian	—
	z	meter	L
Differential length element	dl	meter	L
Differential surface element	dS	square meter	L^2
Differential volume element	dv	cubic meter	L^3
Directivity	D	—	—
Displacement current	I_d	ampere	$T^{-1}Q$
Displacement flux density	$\mathbf{D}$	coulomb/square meter	$L^{-2}Q$
Distance	R, d	meter	L
Drift velocity	$\mathbf{v}_d$	meter/second	LT^{-1}
Electric dipole moment	$\mathbf{p}$	coulomb-meter	LQ
Electric energy	W_e	joule	ML^2T^{-2}
Electric energy density	w_e	joule/cubic meter	$ML^{-1}T^{-2}$
Electric field intensity	$\mathbf{E}$	volt/meter	$MLT^{-2}Q^{-1}$
Electric potential	V	volt	$ML^2T^{-2}Q^{-1}$
Electric susceptibility	χ_e	—	—
Electron density	N	(meter)$^{-3}$	L^{-3}
Electronic charge	e	coulomb	Q
Electronic polarizability	α_e	farad-(meter)^2	$M^{-1}T^2Q^2$
Energy	W	joule	ML^2T^{-2}
Energy density	w	joule/cubic meter	$ML^{-1}T^{-2}$
Force	$\mathbf{F}$	newton	MLT^{-2}
Force per unit volume	$\mathbf{f}$	newton/cubic meter	$ML^{-2}T^{-2}$
Frequency	f	hertz	T^{-1}

TABLE A.1. Cont'd

Quantity	*Symbol*	*Unit*	*Dimensions*
Gravitational field intensity	$\mathbf{g}$	meter/(second)^2	LT^{-2}
Group velocity	v_g	meter/sec	LT^{-1}
Guide impedance	η_g	ohm	$ML^2T^{-1}Q^{-2}$
Guide wavelength	λ_g	meter	L
Impedance	$\bar{Z}$	ohm	$ML^2T^{-1}Q^{-2}$
Incremental relative permeability	μ_{ir}	—	—
Inductance	L	henry	ML^2Q^{-2}
Inductance per unit length	$\mathcal{L}$	henry/meter	MLQ^{-2}
Internal impedance	$\bar{Z}_i$	ohm	$ML^2T^{-1}Q^{-2}$
Internal inductance	L_i, L_{int}	henry	ML^2Q^{-2}
Internal inductance per unit length	$\mathcal{L}_i$	henry/meter	MLQ^{-2}
Intrinsic impedance	η	ohm	$ML^2T^{-1}Q^{-2}$
Intrinsic impedance of free space	η_0	ohm	$ML^2T^{-1}Q^{-2}$
Length	L, l	meter	L
Line charge density	ρ_L	coulomb/meter	$L^{-1}Q$
Linear velocity	$\mathbf{v}$	meter/second	LT^{-1}
Magnetic dipole moment	$\mathbf{m}$	ampere-square meter	$L^2T^{-1}Q$
Magnetic energy	W_m	joule	ML^2T^{-2}
Magnetic energy density	w_m	joule/cubic meter	$ML^{-1}T^{-2}$
Magnetic field intensity	$\mathbf{H}$	ampere/meter	$L^{-1}T^{-1}Q$
Magnetic flux	ψ	weber	$ML^2T^{-1}Q^{-1}$
Magnetic flux density	$\mathbf{B}$	weber/square meter (or tesla)	$MT^{-1}Q^{-1}$
Magnetic polarizability	α_m	(meter)^4/henry	$M^{-1}L^2Q^2$
Magnetic scalar potential	V_m	ampere	$T^{-1}Q$
Magnetic susceptibility	χ_m	—	—
Magnetic vector potential	$\mathbf{A}$	weber/meter	$MLT^{-1}Q^{-1}$
Magnetization surface current density	$\mathbf{J}_{ms}$	ampere/meter	$L^{-1}T^{-1}Q$
Magnetization vector	$\mathbf{M}$	ampere/meter	$L^{-1}T^{-1}Q$
Magnetization volume current density	$\mathbf{J}_m$	ampere/square meter	$L^{-2}T^{-1}Q$
Magnetizing field	$\mathbf{B}_m$	weber/square meter (or tesla)	$MT^{-1}Q^{-1}$
Mass	M, m	kilogram	M
Mobility of electron	μ_e	(meter)^2/volt-second	$M^{-1}TQ$
Mobility of hole	μ_h	(meter)^2/volt-second	$M^{-1}TQ$
Molecular polarizability	α	farad-(meter)^2	$M^{-1}T^2Q^2$
Mutual inductance	L_{12}, L_{21}	henry	ML^2Q^{-2}
Mutual inductance per unit length	$\mathcal{L}_{12}, \mathcal{L}_{21}$	henry/meter	MLQ^{-2}
Natural frequency of oscillation	f_n	hertz	T^{-1}
Normalized admittance	$\bar{y}$	—	—
Normalized impedance	$\bar{z}$	—	—
Normalized radiation intensity	U_n	—	—
Normalized reactance	x	—	—

TABLE A.1 Cont'd

Quantity	*Symbol*	*Unit*	*Dimensions*
Normalized resistance	r	—	—
Normalized susceptance	b	—	—
Permeability	μ	henry/meter	MLQ^{-2}
Permeability of free space	μ_0	henry/meter	MLQ^{-2}
Permittivity	ϵ	farad/meter	$M^{-1}L^{-3}T^2Q^2$
Permittivity of free space	ϵ_0	farad/meter	$M^{-1}L^{-3}T^2Q^2$
Phase constant	β	radian/meter	L^{-1}
Phase refractive index	μ	—	—
Phase velocity	v_p	meter/second	LT^{-1}
Plasma frequency	f_N	hertz	T^{-1}
Polarization current density	$\mathbf{J}_p$	ampere/square meter	$L^{-2}T^{-1}Q$
Polarization surface charge density	ρ_{ps}	coulomb/square meter	$L^{-2}Q$
Polarization vector	$\mathbf{P}$	coulomb/square meter	$L^{-2}Q$
Polarization volume charge density	ρ_p	coulomb/cubic meter	$L^{-3}Q$
Polarizing electric field	$\mathbf{E}_p$	volt/meter	$MLT^{-2}Q^{-1}$
Position vector of field point	$\mathbf{r}$	meter	L
Position vector of source point	$\mathbf{r}'$	meter	L
Power	P	watt	ML^2T^{-3}
Power density	p	watt/square meter	MT^{-3}
Power dissipation density	p_d	watt/square meter	MT^{-3}
Poynting vector	$\mathbf{P}$	watt/square meter	MT^{-3}
Propagation constant	$\bar{\gamma}$	complex neper/meter	L^{-1}
Propagation vector	$\boldsymbol{\beta}$	radian/meter	L^{-1}
Quality factor	Q	—	—
Radian frequency	ω	radian/second	T^{-1}
Radiated power	P_{rad}	watt	ML^2T^{-3}
Radiation intensity	U	watt/steradian	ML^2T^{-3}
Radiation resistance	R_{rad}	ohm	$ML^2T^{-1}Q^{-2}$
Reactance	X	ohm	$ML^2T^{-1}Q^{-2}$
Reflection coefficient	Γ	—	—
Relative permeability	μ_r	—	—
Relative permittivity	ϵ_r	—	—
Reluctance	$\mathcal{R}$	ampere-turn/weber	$M^{-1}L^{-2}Q^2$
Resistance	R	ohm	$ML^2T^{-1}Q^{-2}$
Resistance per unit length	$\mathcal{R}$	ohm/meter	$MLT^{-1}Q^{-2}$
Skin depth	δ	meter	L
Skin effect resistance	R_s	ohm	$ML^2T^{-1}Q^{-2}$
Skin effect resistance per unit length	$\mathcal{R}_s$	ohm/meter	$MLT^{-1}Q^{-2}$
Solid angle	Ω	steradian	—
Spherical coordinates	r, r_s	meter	L
	θ	radian	—
	ϕ	radian	—
Surface	S	square meter	L^2
Surface charge density	ρ_s	coulomb/square meter	$L^{-2}Q$
Surface current density	$\mathbf{J}_s$	ampere/meter	$L^{-1}T^{-1}Q$

TABLE A.1. Cont'd

Quantity	*Symbol*	*Unit*	*Dimensions*
Susceptance	B	mho	$M^{-1}L^{-2}TQ^2$
Time	t	second	T
Unit normal vector	$\mathbf{i}_N, \mathbf{i}_n$	—	—
Velocity of light in free space	c	meter/second	LT^{-1}
Voltage	V	volt	$ML^2T^{-2}Q^{-1}$
Voltage standing wave ratio	VSWR	—	—
Voltage transmission coefficient	τ_v	—	—
Volume	V	cubic meter	L^3
Volume charge density	ρ	coulomb/cubic meter	$L^{-3}Q$
Volume current density	$\mathbf{J}$	ampere/square meter	$L^{-2}T^{-1}Q$
Wavelength	λ	meter	L
Work	W	joule	ML^2T^{-2}

We know that the dimension of v is LT^{-1}. Hence, we have to show that the dimension of $1/\sqrt{\mu_0\epsilon_0}$ is also LT^{-1}. To do this, we note from Coulomb's law that

$$\epsilon_0 = \frac{Q_1Q_2}{4\pi FR^2}$$

Hence, the dimension of ϵ_0 is $Q^2/[(MLT^{-2})(L^2)]$ or $M^{-1}L^{-3}T^2Q^2$. We note from Ampere's force law applied to two infinitesimal current elements parallel to each other and normal to the line joining them that

$$\mu_0 = \frac{4\pi FR^2}{(I_1\,dl_1)(I_2\,dl_2)}$$

Hence, the dimension of μ_0 is $[(MLT^{-2})(L^2)]/(QT^{-1}L)^2$ or MLQ^{-2}. We now obtain the dimension of $1/\sqrt{\mu_0\epsilon_0}$ as $1/\sqrt{(M^{-1}L^{-3}T^2Q^2)(MLQ^{-2})}$ or LT^{-1}, which is the same as the dimension of v. It should however be noted that the test for the equality of the dimensions of the two sides of a derived equation is not a sufficient test to establish the equality of the two sides since any dimensionless constants associated with the equation may be in error.

It is not always necessary to refer to the table of dimensions for checking the possible validity of a derived equation. For example, let us assume that we have derived the expression for the characteristic impedance of a transmission line, i.e., $\sqrt{\mathcal{L}/\mathcal{C}}$ and we wish to verify that $\sqrt{\mathcal{L}/\mathcal{C}}$ does indeed have the dimension of impedance. To do this, we write

$$\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{\omega\mathcal{L}1}{\omega\mathcal{C}1}} = \sqrt{\frac{\omega L}{\omega C}} = \sqrt{(\omega L)\left(\frac{1}{\omega C}\right)}$$

We now recognize from our knowledge of circuit theory that both ωL and $1/\omega C$, being the reactances of L and C, respectively, have the dimensions of impedance. Hence, we conclude that $\sqrt{\mathcal{L}/\mathcal{C}}$ has the dimension of $\sqrt{(\text{impedance})^2}$ or impedance.