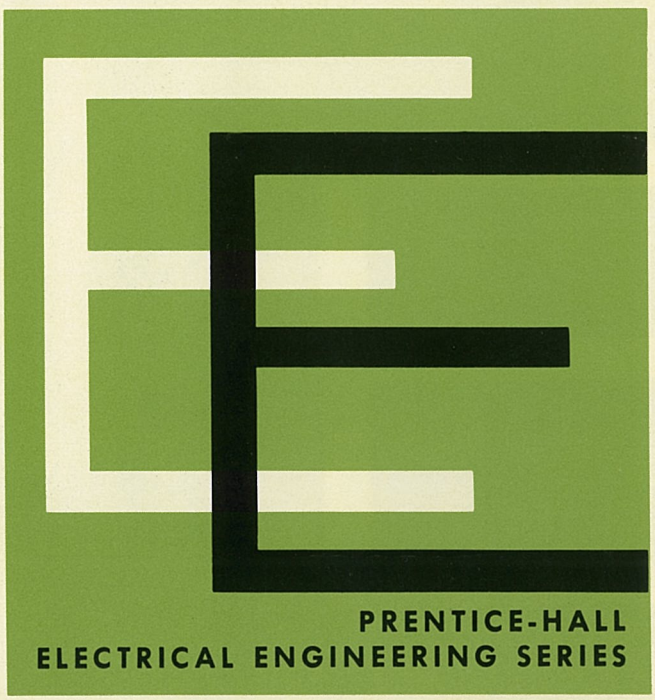
N. NARAYANA RAO

Basic Electromagnetics With Applications

MICROWAVES AND FIELDS SERIES

Nathan Marcuvitz,
Editor



BASIC ELECTROMAGNETICS WITH APPLICATIONS

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Pearson Education, Inc., has transferred the rights of this 1972 Prentice-Hall edition to the author in January 2018. The author has decided to make the book available on the web free of charge to the teachers, students, and others all over the world for the purpose of teaching and learning the fundamentals of electromagnetics. Any use involving monetary transaction violates the spirit of this decision.

There is a Subhashitam (Worthy Saying) in Sanskrit, which says:

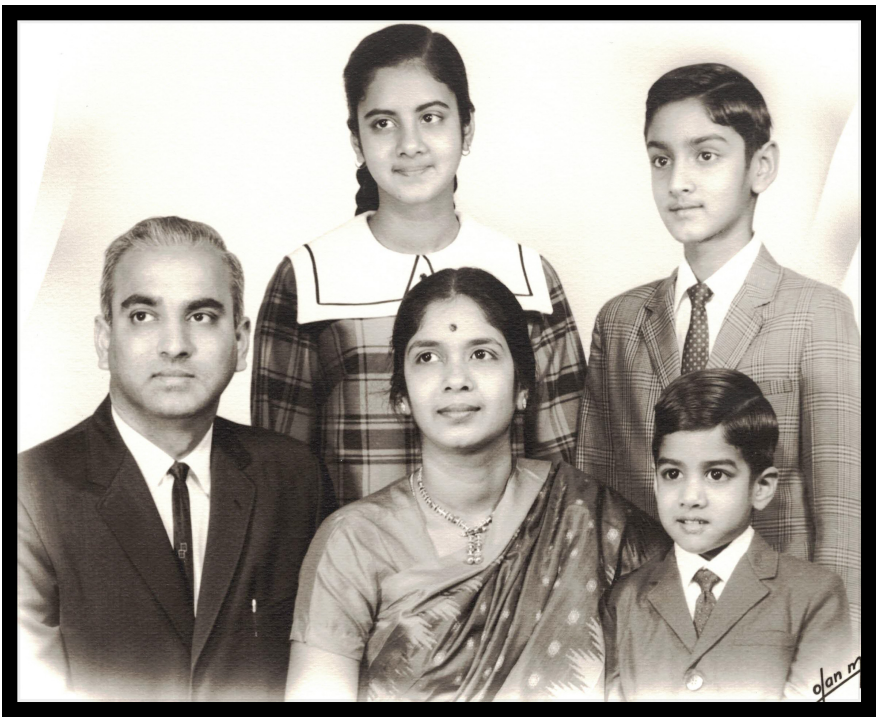
Annadaanam param daanam
Vidyadaanam atahparam
Annena kshanikam triptih yaavajjiivamcha vidyayaa.

The gift of food is a great gift
Greater still is the gift of knowledge
While food provides a momentary contentment, knowledge provides a lasting fulfillment.

This "webook (web + book)" constitutes the gift, by the author and his department, of the knowledge of the subject of electromagnetics, based on Maxwell's equations, which "today underpin all modern information and communication technologies."

To

My Wife and Children
Sarojini and Vanaja, Durgaprasad, and Hariprasad



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Preface

This book is designed for an introductory undergraduate course in electromagnetics. In view of the rapid growth of the several specialized branches of electrical engineering, a student may not have the opportunity to take advanced courses in field theory. On the other hand, electromagnetics is one of the fundamental subjects having a wide variety of applications, as evidenced by its place in the undergraduate core curriculum. Hence, a thorough understanding of the basic concepts of electromagnetics must be imparted to the electrical engineering student in the introductory course itself.

To facilitate the aforementioned task, an attempt is made in this book to present the basic field theory at an introductory level and at the same time in sufficient depth to establish the concepts firmly in the student's mind and to enable the interested student to use the advanced books without having to relearn the subject or reorient his understanding of the concepts. This is done by combining the classical approach of introducing field theory with statics and the modern approach of emphasizing dynamics to develop Maxwell's equations and the associated constitutive relations in a gradual manner and finally use them to discuss several applications. A number of worked-out examples are distributed throughout the book to illustrate and, in some cases, extend the various concepts and to aid the student's grasp of the subject matter.

The book does not presuppose knowledge of vector analysis. Chapter 1 contains the discussion of coordinate systems and vector analysis necessary and sufficient for the remaining chapters. Other mathematical tools such as the Dirac delta function and the phasor technique are introduced wherever necessary.

Chapters 2 and 3 are devoted to static electric and magnetic fields, respectively, in free space. Starting with Coulomb's and Ampere's laws in chapters 2 and 3, respectively, Maxwell's equations for static fields are introduced in a logical manner. The coverage of static magnetic field in chapter 3 is as much detailed as the coverage of static electric field in chapter 2 unlike the traditional mode of presentation in which the electric field topics are emphasized.

Chapter 4 is devoted to the electromagnetic field in free space. Maxwell's equations for time-varying fields are introduced. Energy storage in electric and magnetic fields and power flow in electromagnetic field are discussed. The use of phasor technique in dealing with sinusoidally time-varying vector fields is illustrated. Maxwell's equations and the power and energy relations are then specialized for sinusoidally time-varying fields.

The discussion in chapters 2, 3, and 4 is in terms of the field vectors **E** and **B**. Chapter 5 is devoted to the study of fields in the presence of materials. The interaction between fields and charges in materials is discussed in terms of equivalent charge and current distributions which are related to the fields and act as though they were situated in free space, thereby entering into Maxwell's equations. By defining field vectors **D** and **H** and relating them to **E** and **B**, respectively, Maxwell's equations for free space developed in chapters 2, 3, and 4 are generalized so that they can be used for material media as well as for free space. The power and energy relations developed in chapter 4 are also generalized for material media. Boundary conditions are derived for the fields.

Chapter 6 serves as an introduction to the applications of Maxwell's equations. A variety of topics providing a continuous coverage from statics to electromagnetic waves via quasistatics and distributed circuits are discussed. The presentation is oriented towards introducing the fundamental concepts leading to and associated with the applications. For example, the circuit parameters conductance, capacitance, and inductance are introduced simultaneously so that the student can better appreciate the development of the frequency behavior of a physical structure made up of two parallel conductors leading to the concept of a distributed circuit. Yet another example is the introduction of waveguides by starting with uniform plane waves incident obliquely on a perfect conductor, which provides a physical understanding of the waveguiding phenomenon.

There is enough material in this book for a two-semester course. However, by deemphasizing certain topics and omitting certain other topics, it is possible to use this text for a one-semester course. In the latter case, the student can read the remaining material by himself with the aid of the answers to the odd-numbered problems included at the end of the book. The many example problems throughout and the numerous homework problems at the end of each chapter make this book especially suitable for a course oriented towards problem solving.

This text is based on lecture notes prepared for courses taught since 1965 at the University of Illinois at Urbana-Champaign and earlier at the University of Washington. I am indebted to Professor E. C. Jordan at the University of Illinois and Professor A. V. Eastman at the University of Washington for their help in several instances without which this book would not have materialized.

Urbana, Illinois *N. Narayana Rao*