University of Illinois Urbana-Champaign
Department of Aerospace Engineering
Fall 2026

AE 498 ARM/ARO – Advanced Review of Basic Mathematics

4 credit hours

This syllabus is not an exhaustive description of all details of the course. Students are free to contact the instructor with any additional questions or concerns at any time.

Instructor

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Contact: most easily by e-mail, and then we can arrange in person

Teaching Assistant

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Office hours: To be agreed at the beginning of semester

Lecture Times and Organization

Lectures will generally be held on *MW 9-10:20pm* in *106B3 Engineering Hall*. Temporary adjustments are possible, and even likely; some are already almost certain. Attendance is *not* mandatory: students are welcome to adjust their course experience to their learning style, as long as doing so does not disturb learning styles of the others. **The intention is that all the lectures will be recorded and made available to all students: while the focus of that delivery mechanism is on students taking the course online, all students may equally make use of the recordings.** Especially keeping this capability in mind, students are required to engage in appropriate behavior to protect the health and safety of the community.

Additional optional review sessions may be held at key points of the semester, i.e., at the ends of large chapters or prior to milestone assessments. These will be more informal and held at non-standard times to attract a larger number of participants, but effort will be taken to make them accessible to both online and in-person students.

Course materials (notes, homeworks), as well as grades, will be delivered through the course's **Canvas page**. Students should make sure to be enrolled in the page, and be set up to quickly receive Canvas announcements. *All important announcements will be communicated to all students over Canvas*.

Course Description

This is a mathematics course. Its goal is to provide a rigorous foundation to the broad variety of techniques, topics, and areas of mathematics that students have almost invariably previously seen during their undergraduate careers. In that sense, the course provides little entirely novel material: instead, its *treatment* of material is potentially novel, especially to students coming from applied engineering backgrounds.

The emphasis of the course is naturally on topics and tools that are expected to loom large in the students' graduate career. *Linear algebra* and *real analysis* — in undergraduate courses often known under the less rigorous moniker of *calculus* — are certainly the star characters. In the first half of the semester, after providing the definitions for the fundamental notions of sets and functions, we will focus on exploring the connection between that deceptively simple central object of linear algebra — a *matrix* — and its true meaning: a *linear operator* between two vector spaces. This investigation will first require us to formally define the notion of a vector space and a linear functions, and the corresponding notions of a basis and a span. A vector space may come armed with additional bells and whistles that are often natural in the spaces that the students have seen previously, *but not in all vector* spaces, so we will consider the notions of scalar product, norm, distance, and consequently orthonormality. We will then proceed to introduce the matrix as a representation of a linear operator for particular bases, correspondingly define a similarity transform and the notion of an isomorphism, and consider *why* matrices are multiplied in the way they are.

Following our introduction of a matrix, we will provide a rigorous treatment of the crucial elements of matrix — i.e., operator — structure: eigenvalues and eigenvectors, diagonalization, Jordan form, and SVD decomposition. To connect this material with standard applied topics of optimization and machine learning, we will briefly discuss linear programming, projections, and least squares optimization.

The second half of the course will be dedicated to real analysis. The central object that we will examine in that portion is the notion of a *derivative*. In that context, the first question that we will ask ourselves will be: when does a function admit a derivative? Answering this problem will require us to formally define continuity, as well as explore the ways in which a function may or may not converge to a given value. After defining the derivative — which we will first do for single-variable scalar-valued functions before extending it to more a more general setting — we will see how the set of iterated derivatives of a function at a given point may, *but might not*, provide all the information about the function throughout its domain. In other words, we will discuss Taylor series, and to provide a counterpoint which does not use derivatives, widely applicable Fourier series. Of course, discussing these will require us to define the notion of a series, and discuss its conditional and unconditional convergence. We will end by a short discussion on a topic which brings together lessons from both linear algebra and real analysis: that of linear ordinary differential equations.

Throughout the course, in parallel with our treatment of specific mathematical topics, we will make an active effort to promote *mathematical literacy*: proper use of notation and basic formal logic, clear and rigorous proof-writing, and conscientious use of literature and prior results.

Assignments and Grading

The deliverables for the course will consist of 5 homework assignments, an in-person (or remotely proctored, for students who are not on campus) midterm, and a final exam. The weights for the deliverables will be distributed as follows:

Homework 1: 10%
Homework 2: 10%
Midterm: 20%
Homework 3: 10%
Homework 4: 10%
Homework 5: 10%
Final exam: 30%

Additional extra credit may be offered during the semester, but should not be counted on.

The final grades for the course will be calculated by the following formula: A-/A/A+ = 90-100, B-/B/B+ = 80-89.99, C- /C/C+ = 70-79.99, D-/D/D+ = 60-69.99, F = 0-59.99, where the "-" modifier will be assigned to those grades with the unit digit 0-1 (e.g., 91.87 = A-) and "+" modifier to those grades with the unit digit 8-9 (e.g., 78.02 = C+). The grades will not be rounded up, rounded down, nor "curved".

Form and Submission of Deliverables

Written submissions of the homework assignments and the final project will be due at **11:59pm** (Central time) of the deadline dates indicated by the instructor (see tentative course outline for predicted dates). Late submission of a particular deliverable, if not agreed with the instructor, will be penalized at the rate of 15% of the weight of the deliverable per day (prorated for the actual delay time; e.g., a 2-hour delay incurs a penalty of 1.25% of the total value of the deliverable).

Assignments should be submitted to the course Canvas page. Students are responsible for timely submission of the assignments. If there are issues with the page, students are welcome to submit the assignments to the instructor by e-mail.

The midterm will be in person or, for those students who will need to take it remotely, in a timed classroom-like setting. It will be 80 minutes in length.

All deliverables, and particularly the midterm and the final exam will largely follow the material covered in the lectures. The students will be able to use any books, computers, or notes during any of the homeworks, the midterm, and the final exam. Students will also be able to communicate with each other or other persons while working on any homeworks, *but not the midterm and the final exam*. However, they will need to acknowledge who they worked with, and will naturally need to write their own answers. **In the context of this course, we will treat generative AI as *a person*** and impose the same rules as in the preceding sentences.

For any deliverable, students may also possibly need to answer additional questions from the instructor about the deliverable material, which might cause a change in the deliverable mark.

Emergencies, injuries, mental health issues do happen; when faced with obstacles to proceeding in class along the regular plan – and such obstacles will almost certainly arise, given the current state of flux of the society – students should contact the instructor for any modifications to the deliverable schedules.

Prerequisites and Literature

There is no required text for the course. With possible small exceptions intended for independent study, all new topics required for success in the course will be discussed during the lectures. *In terms of graduate courses*, this course is fairly introductory, and there are no official prerequisites. However, the students realistically ***must*** **have taken their undergrad institution's linear algebra and calculus sequences**: this course may prove almost impossible to traverse without prior exposure and success in those courses.

The course material will broadly follow four books:

- *Mathematical Analysis I* by Elias Zakon (very briefly)
- *Linear Algebra Done Right* by Sheldon Axler
- *Convex Optimization* by Stephen Boyd and Lieven Vandenberghe (briefly)
- *Elementary Real Analysis* by Brian S. Thomson, Andrew M. Bruckner, and Judith B. Bruckner

All of these books have been freely released online by their authors/publishers. The course deliverables may explicitly refer to any book at any point.

Students are not required to purchase any textbooks or other materials.

Academic Integrity

Students are welcome to use any literature and work together on their homework assignments. They are, however, required to:

1) write submissions on their own,
2) mention any peers they were working and any online or offline sources that they used, and
3) respond to any subsequent questions on the material posed by the instructor, whether in person or over e-mail. The answers to the instructor's questions may affect the assigned mark.

Because of the nature of the midterm and the final exam, students are expected to work alone on it, while making use of literature. Students are required to familiarize themselves with the University's Academic Integrity Policy and Procedure, available at http://studentcode.illinois.edu/article1/part4/1-401/, and abide by that policy in full.

Respect and Growth in the Classroom

The effectiveness of education is dependent upon creating a safe and encouraging environment that allows for the open exchange of ideas. In that regard, growing as a whole person is far more important than growing in technical knowledge or a scientific discipline, and includes growing in respect and understanding of others. While everyone has different personalities and perspectives and it takes time to adapt to one another, maintaining an environment where everyone can freely and joyfully contribute is one of the explicit goals of the class.

The Aerospace Engineering department has a committee called Aero's Space to Belong. Among their other activities, they offer a reporting process. In case of conflict that undermines anyone's sense of respect, self-respect, or belonging, every student should consider using the committee's resources: https://aerospace.illinois.edu/community.

Accommodations

To obtain disability-related academic adjustments and/or auxiliary aids, students that require special accommodations must contact the instructor and the Disability Resources and Educational Services (DRES) as soon as possible. Students are welcome to contact the instructor at any time with any accommodation-related needs. To contact DRES, visit 1207 S. Oak St., Champaign, call 217-333-4603 (V/TTY), e-mail a message to disability@illinois.edu, or visit https://dres.illinois.edu. If a student is concerned that they have a disability-related condition that is impacting their academic progress, there are academic screening appointments available that can help diagnosis a previously undiagnosed disability. These may be accessed by visiting the DRES website.

Illinois law requires the University to reasonably accommodate its students' religious beliefs, observances, and practices in regard to admissions, class attendance, and the scheduling of examinations and work requirements. If there is a conflict between course deadlines and any religious observances, students should notify the instructor and follow the procedure at https://odos.illinois.edu/community-of-care/resources/students/religious-observances/ to request appropriate accommodations. These steps should be conducted in the first two weeks of classes.

Privacy and Reporting

The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University's Title IX Office. In turn, an individual with the Title IX Office will provide information about rights and options, including accommodations, support services, the campus disciplinary process, and law enforcement options. A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality, can be found here: https://wecare.illinois.edu/resources/students. Other information about resources and reporting is available here: https://wecare.illinois.edu.

Any student who has suppressed their directory information pursuant to Family Educational Rights and Privacy Act (FERPA) should self-identify to the instructor to ensure protection of the privacy of their attendance in this course. See https://registrar.illinois.edu/academic-records/ferpa/ for more information on FERPA.

Modifications to the Syllabus

The instructor reserves the right to modify any and all parts of this syllabus throughout the semester. All modifications will be made solely in the interest of time scheduling, beneficial adaptation to changing public health circumstances, accurate measurement of the students' success, and improvement of the students' educational outcomes. Any modifications will be transparently communicated to the students.

Course Outline

Every element of this section is **tentative**: the mentioned topics and times for homeworks and the midterm serve only as a guideline.

Week	Topic	Day 1	Day 2
1: Aug 24-28		No class	No class
2: Aug 31 – Sep 4	set, set relations, set operations, introduction to proofs, quantifiers, relations, equivalence, functions, image, preimage, invertibility, mathematical induction	Regular class	Regular class
3: Sep 7-11	vector space	**Labor Day**	Regular class
4: Sep 14-18	vector subspace, linear map, kernel, image, linear combination, spanning set, linear independence, basis, dimension, inner product, Cauchy-Schwarz, norm, triangle inequality	Regular class (x2?) **Homework 1 given**	Regular class (x2?)
5: Sep 21-25	distance, matrix as a representation of a linear map, composition of linear maps, rank	Regular class **Homework 1 due**	Regular class **Homework 2 given**

6: Sep 28 – Oct 2	linear map invertibility, isomorphism, invariant subspaces, eigenvalues, minimal polynomial	Likely no class	Regular class **Homework 2 due**
7: Oct 5-9	eigenvectors, characteristic polynomial, diagonalizability, eigenvalues without determinants	Regular class **Homework 3 given**	Regular class
8: Oct 12-16	nilpotent operators, Jordan form, singular values	Regular class **Homework 3 due**	Midterm discussion (x2?)
9: Oct 19-23		**Midterm**	Midterm review
10: Oct 26-30	optimization, minimum vs. infimum, linear programming, examples of linear programs, simplex method	Regular class	Regular class
11: Nov 2-6	knapsack problem, Chebyshev approximation problem, linear least squares	Regular class	Regular class **Homework 4 given**
12: Nov 9-13	epsilon-delta notation, continuity, properties of continuous functions, differentiability, one-sided derivatives, squeeze theorem, properties of the derivative, Lipschitz functions	Regular class	Regular class **Homework 4 due**
13: Nov 16-20	first-order conditions for optimization, Rolle's theorem, mean value theorem, monotonicity from derivatives, series, conditional and absolute convergence, convergence tests, Taylor series	Regular class	Regular class **Homework 5 given**
--: Nov 23-27		**Fall Break**	
14: Nov 31 – Dec 4	products and compositions of Taylor series, Fourier series, uniform convergence, multivariate derivatives, systems of linear ordinary differential equations	Regular class	Regular class **Homework 5 due**
15: Dec 7-11		Final exam discussion	Final exam discussion
Dec 14, 8am	**Final exam**		