

TAM 541 – Mathematical Methods I

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Lecture: MW 11:00am - 12:50pm, LuMEB 4100

Outline of Topics

I. Complex Numbers and Complex Analysis

- Complex numbers, complex plane, polar form, algebraic operations
- Functions of a complex variable, elementary functions and their properties
- Complex differential calculus, analyticity, Cauchy-Riemann equations
- Conformal mapping, bilinear transforms, applications
- Complex integral calculus, Cauchy's theorem, Taylor and Laurent series, residue theorem

II. Linear Algebra

- Vectors, vector spaces, norm and products, span and subspace, generalized vector spaces
- Linear dependence, dimensions, bases, Gram-Schmidt orthogonalization, least squares approximation
- Matrices, systems of linear equations, inverse, determinant, eigenvalue problems, Jordan normal form, applications
- Tensor algebra, index notation, coordinate transformations; tensor products

III. Ordinary Differential Equations

- Introduction to ODEs: definitions, order, boundary and initial value problems
- ODEs of first order: linear equations, separable equations, exact solutions, integrating factors, applications
- Linear ODEs of second order and higher: linear dependence, homogeneous equations, non-homogeneous equations, systems of ODEs, applications
- Power series solutions: general theorems, Frobenius method, Legendre polynomials, Gamma function, Bessel function
- Laplace transform: properties and applications to ODEs
- Fourier transform: properties and applications
- Elements of numerical solutions: Euler method, midpoint rule, Runge-Kutta methods, stability
- Qualitative methods for nonlinear equations: phase plane, singular points and stability