TAM 416 / Aero 452 Introduction to Nonlinear Dynamics and Vibrations
MW 1-2:50pm CST 403B2 Engineering Hall and on-line, Fall 2026

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Department of Mechanical Science and Engineering

Canvas course site: https://canvas.illinois.edu/courses/72710 (under construction)

Syllabus

We start with an introduction to the basic concepts and methods of the qualitative and quantitative theory of Nonlinear Dynamics and Vibrations. After discussing theorems on the existence and uniqueness of solutions for some general classes of nonlinear oscillators described by ordinary differential equations, we proceed to a description of the dynamics in phase space. We introduce the definitions of dynamical flow, equilibrium points, and periodic, quasiperiodic and chaotic orbits. We then examine basic asymptotic methods for analyzing the free and forced responses of single- and multi-degree-of-freedom nonlinear oscillators, including the methods of Lindstedt-Poincare', averaging, and multiple-scales. In addition, we discuss the concept of slow-fast partitions of the dynamics and the method of complexification-averaging (CX-A) for strongly nonlinear (even non-linearizable) systems. We continue with a systematic study of forced (fundamental, subharmonic and superharmonic) resonances, internal resonances and combination resonances that encountered in broad classes of dynamical systems, together with a discussion of linearized stability analysis. This leads to a detailed discussion and study of linearized stability, Floquet theory, the theory of linear parametrically excited systems, and the notion of parametric resonance. Examples are given with the Mathieu, Hill and Mathieu-Hill oscillators, leading to the derivation of stability diagrams in their corresponding parameter planes. We then provide a detailed discussion of time-discrete dynamical systems (nonlinear maps) and show how this class of dynamical systems can be used to study the global dynamics of low dimensional nonlinear oscillators. The concept of Poincare' map is introduced in detail, and systematic techniques for its numerical computation are discussed. We then proceed to an introduction of the concepts of nonlinear normal mode, nonlinear localization and targeted energy transfers in systems of coupled nonlinear oscillators. We conclude with a brief introduction to nonlinear vibrations of elastic continua with applications to nonlinear beam theory. Numerous applications of nonlinear dynamics and vibrations in mechanics and engineering are presented throughout the course.

Course organization

This course has an on-line component, so both on-campus and off-campus students are registered. Attendance of the lectures by the on-campus students is <u>mandatory</u>, while the off-campus students should visit the canvas course site as well as the following site for the lecture recordings:
https://mediaspace.illinois.edu/channel/channelid/414714522
You should be able to access the course web site with your Network ID and password. The IT contact person for the on-line component of the course is Sara Smith who should be contacted for questions or on-line assistance. Her contact details are:

Sara Smith (she/her)
Office Support Associate
Office of Online & Professional Engineering Programs
Grainger College of Engineering, University of Illinois
217-244-3611, ssmith19@illinois.edu

Office hours of the instructor will be announced once the semester starts after consultation with the students. Class material, including announcements, lecture notes, homework (HW) postings and solutions to the HWs, will be uploaded to the course canvas site. Please make sure to visit that site often to remain updated on course developments.

There will be a regular series of HW assignments which will be assigned from a HW file and posted on Canvas. For on-campus students, HW solutions are due in class on the due dates specified. Off-campus students should submit their HW solutions in pdf format directly to the TA while cc-ing me, on the specified due dates. Due to logistical constraints and reasons of fairness to your fellow students, no late HW solutions will be accepted and late HWs won't be given any credit, unless there is a documented serious reason (e.g., medical or family). You are encouraged to collaborate and cooperate with your peers on the HW assignments; however, you should hand in your own original HW solutions. To get full credit, your HW solutions need to show great attention to detail, outline all steps of your derivations and be written in a systematic, neat, organized and clear way; moreover, if the tasks involve numerical computations you'll need to include a list of your codes (e.g., Matlab, Python or any other software tool of your choice), and any computer-generated plots need to show attention to detail, e.g., there should be labels on the axes, correct depiction of vectors, etc.

The Final Exam for this course is scheduled for **Wednesday, December 16, 7:00-10:00pm, 3100 LUMEB** for on-campus students, and on the same day for the on-line students (please coordinate with Ms. Sara Smith for your exam). Please take a note of this date and time in your schedules since no conflict exams will be given.

Attendance policy (for on campus students)

Attendance of lectures is mandatory for on-campus students, so missing a total of more than four lectures during the semester will result in getting no credit in class participation. So, please make sure to sign in your name on the attendance sheet at the beginning of each lecture.

Total grade breakdown

Class participation:	5%
Homework:	45%
Final Exam:	50%

Recommended (not required) references

Nayfeh, A.H., and Mook, D., 1985, *Nonlinear Oscillations*, Wiley Interscience, New York
Rand, R.H., 2001, *Lecture Notes on Nonlinear Vibrations*, http://hdl.handle.net/1813/28989
Verhulst, F., 1990, *Nonlinear Differential Equations and Dynamical Systems*, Springer Verlag

Prerequisites

Basic theory of ordinary differential equations and linear algebra, basic linear dynamics and vibrations

Anti-racism and inclusivity statement

This is an equal opportunity classroom environment. We value the diversity represented by the participants in this course. Our diversity is a primary source of ideas and perspectives. As you work through the course, practice using this diversity to your advantage.

Sexual misconduct reporting obligation

The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University's Title IX Office. In turn, an individual with the Title IX Office will provide information about rights and options, including accommodations, support services, the campus disciplinary process, and law enforcement options.

A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality, can be found here: wecare.illinois.edu/resources/students/#confidential.

Other information about resources and reporting is available here: wecare.illinois.edu.

Academic integrity

The University of Illinois at Urbana-Champaign Student Code should also be considered as a part of this syllabus. Students should pay particular attention to Article 1, Part 4: Academic Integrity. Read the Code at the following URL: http://studentcode.illinois.edu/.

Academic dishonesty may result in a failing grade. Every student is expected to review and abide by the Academic Integrity Policy: https://studentcode.illinois.edu/article1/part4/1-401/. Ignorance is not an excuse for any academic dishonesty. It is your responsibility to read this policy to avoid any misunderstanding. Do not hesitate to ask the instructor(s) if you are ever in doubt about what constitutes plagiarism, cheating, or any other breach of academic integrity.

Religious observances

Illinois law requires the University to reasonably accommodate its students' religious beliefs, observances, and practices in regard to admissions, class attendance, and the scheduling of examinations and work requirements. You should examine this syllabus at the beginning of the semester for potential conflicts between course deadlines and any of your religious observances. If a conflict exists, you should notify your instructor of the conflict and follow the procedure at https://odos.illinois.edu/community-of-care/resources/students/religious-observances/ to request appropriate accommodations. This should be done in the first two weeks of classes.

Disability-related accommodations

To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail disability@illinois.edu or go to https://www.disability.illinois.edu. If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available that can help diagnosis a previously undiagnosed disability. You may access these by visiting the DRES website and selecting "Request an Academic Screening" at the bottom of the page.

Family educational rights and privacy act (FERPA)

Any student who has suppressed their directory information pursuant to Family Educational Rights and Privacy Act (FERPA) should self-identify to the instructor to ensure protection of the privacy of their attendance in this course. See https://registrar.illinois.edu/academic-records/ferpa/ for more information on FERPA.

Note regarding public safety

Emergencies can happen anywhere and at any time, so it's important that we take a minute to prepare for a situation in which our safety could depend on our ability to react quickly. Take a moment to learn the different ways to leave this building. If there's ever a fire alarm or something like that, you'll know how to get out and you'll be able to help others get out. Next, figure out the best place to go in case of severe weather – we'll need to go to a low-level in the middle of the building, away from windows. And finally, if there's ever someone trying to hurt us, our best option is to run out of the building. If we cannot do that safely, we'll want to hide somewhere we can't be seen, and we'll have to lock or barricade the door if possible and be as quiet as we can. We will not leave that safe area until we get an Illini-Alert confirming that it's safe to do so. If we can't run or hide, we'll fight back with whatever we can get our hands on. If you want to better prepare yourself for any of these situations, visit police.illinois.edu/safe. Remember you can sign up for emergency text messages at emergency.illinois.edu.