BIOE 485 Computational Mathematics for Machine Learning and Imaging

A one-semester, four-credit-hour course offered at the Grainger College of Engineering by the Department of Bioengineering at the University of Illinois at Urbana-Champaign.

Instructors

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Please email me for meeting appointments

Course Prerequisites

Senior undergraduate or graduate standing in an engineering degree program or consent of instructor.

Course Description

The course will cover fundamental mathematical and computational methods needed to implement computational imaging and machine learning solutions. The course will introduce:

- Fundamental objects and tools relevant to linear algebra, vector spaces, and matrix decompositions;
- Numerical optimization methods that represent core components of computational imaging and machine learning. Basic concepts and tools in vector calculus will be first introduced, including gradients of vector-valued functions and matrices, as well as backpropagation and automatic differentiation. Then, optimization-based formulations of computational imaging and machine learning problems will be covered. Afterwards, numerical optimization techniques will be introduced in detail with an emphasis on first-order deterministic and stochastic gradient-based methods;
- Fundamental concepts in probability theories, and basic techniques in statistical inference such as Bayesian inference, approximate inference, as well as stochastic sampling methods;
- Applications in computational imaging and machine learning, including classification, regression, dimensionality reduction, and density estimation.

Student Learning Objectives (SLOs)

Upon completion of this course, the students will:

- Understand the mathematical concepts and foundations needed for computational imaging and machine learning
- Demonstrate the ability to formulate computational imaging and machine learning problems from the statistical inference and/or numerical optimization perspectives;
- Apply computational tools to solve computational imaging and machine learning problems;
- Gain hands-on experience of implementing computational tools to solve simulated and/or practical computational imaging and machine learning problems.

Required Resources

- *Mathematics for Machine Learning*, M. Deisenroth, A. Faisal, and C. Ong, Cambridge University Press.
- Additional reading material will be assigned from a combination of book chapters, review articles, and primary research papers.

Assessment and Grading

Homework: 30%

- Homework projects will be assigned based on lectures and assigned reading material.
- Students may discuss homework problems, but students must complete their own work and write up solutions independently.

Machine Problems: 40%

- The machine problems (MPs) or coding assignments are designed to reinforce students' understanding of the course material and develop practical skills in the Python programming language.
- Students may also discuss machine problems, but students must complete their own work and write up code independently.

Project: 30% (Final project presentation: 10%; Final project report and code: 20%)

- The course will include a capstone project that involves the application of numerical optimization methods to solve a problem in biomedical image computing. The addressed problem can relate to an inverse problem in image formation or training of a machine learning algorithm. Students can work in groups that can contain up to three students. The projects will involve the implementation and systematic characterization and comparison of competing numerical optimization methods. The accuracy and computational efficiency of the methods will be assessed. Motivations for the chosen optimization methods should be provided. Each group will be granted an allocation of computational time on the department computer cluster. Towards the end of the course, each team will present their project in a short, roughly ten-minute-long, presentation. Each team will also prepare a final report (between 4 to 8 pages) in the format of a biomedical imaging or machine learning conference paper. The final grade will depend on the rigor of the evaluation studies and quality of the final presentation and report.

Additional Course Polices

In general, homework deadlines are firm. Special circumstances regarding absence or forbearance will be handled on a case-by-case basis at the discretion of the instructor or program director. Please inform the instructor promptly if additional consideration is required.

Academic Integrity

In brief, an infraction of academic integrity is any one of the following:

- Cheating – using or attempting to use unauthorized materials
- Plagiarism – representing the words, work, or ideas of another as your own
- Fabrication – the falsification or invention of any information, including citations
- Facilitation – helping or attempting to help another commit an infraction
- Bribes, Favors, and Threats – actions intended to affect a grade or evaluation
- Academic Interference – tampering, altering or destroying educational material or depriving someone else of access to that material

It is the students' responsibility to refrain from infractions of academic integrity, from conduct that may lead to suspicion of such infractions, and from conduct that aids others in such infractions. "I did not know" is not an excuse. Please ask the instructor for clarification if you are unsure of their expectations.

The complete text of the University of Illinois student code can be found online at http://studentcode.illinois.edu/article1/part4/1-401/. Additional relevant information may be found by searching "academic integrity" at the University of Illinois website (https://illinois.edu).

Students of various Schools, Colleges and Departments within the university may have additional rights, requirements or resources regarding academic integrity so students are encouraged to consult the information specific to their particular program.

Statement of Accessibility & Accommodation

To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail disability@illinois.edu or go to the DRES website. If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available on campus that can help diagnosis a previously undiagnosed disability by visiting the DRES website and selecting "Sign-Up for an Academic Screening" at the bottom of the page.

Expectations for Students

- Participate throughout each week.
- Consider and respect others' opinions. Complete all assignments on time.
- Discuss concerns privately with the instructor and forum moderators.

Tentative Course Schedule of Topics

Dates	Lectures
Week 1 (08/25)	Introduction to the course **Review of linear algebra** Vectors, matrices, transpose, and inverse
Week 2 (09/01)	**Vector spaces, vector geometry** Vector spaces, linear independence, linear mappings Norms, inner products, positive definiteness, orthogonality, projections, and rotations (HW1 released)
Week 3 (09/08)	Solving linear equations (least-squared, minimum-norm solutions), pseudoinverse **Matrix decompositions** Determinants, eigen analysis, EVD
Week 4 (09/15)	SVD and matrix approximation Principal component analysis (HW2 released) **Vector calculus** Gradients of vector-valued functions and matrices Backpropagation algorithm
Week 5 (09/22)	**Introduction to numerical optimization** (MP1 released) Formulations Constrained vs. unconstrained optimization, optimality conditions Convex sets and functions
Week 6 (09/29)	**Unconstrained optimization algorithms** Vanilla gradient descent (HW3 released) Convergence analysis
Week 7 (10/06)	**Accelerated gradient descent** Momentum and Nesterov's method (MP2 released) Newton's method, gradient projection

Week 8 (10/13)	**Stochastic gradient descent (SGD)** Machine learning motivations, basic formulation, mini-batch SGD (HW4 released) **Constrained optimization** Formulation, inequality and equality constraints, penalty functions, barrier functions
Week 9 (10/20)	Soft constraints (regularization), constraint reduction Lagrange multipliers
Week 10 (10/27)	Proximal operators, KKT conditions, duality (HW5 released) **Applications in machine learning: Discriminative learning** Classification and regression **(project proposals due, end of week**)
Week 11 (11/03)	**Review of probability theory** Definitions and axioms of probability, discrete and continuous random variables, probability density and mass functions, Bayes theorem (MP3 released) CDF, PDF, expectation, moment generating functions Marginals, independence, conditional densities, correlation, and covariance
Week 12 (11/10)	Vector random variables, expectations of random vectors, jointly Gaussian random vectors Jensen's, Markov's, Chebyshev's, Chernoff's inequalities, CTL, LLN (HW6 released)
Week 13 (11/17)	**Bayesian inference & sampling-based approximate inference** Maximum likelihood estimation Bayesian estimation
Week 14 (11/24)	**Fall break**
Week 15 (12/01)	Importance sampling, MCMC (MP4 released) **Applications in machine learning: Generative learning** Formulation of density estimation Gaussian mixture models, the EM algorithm
Week 16 (12/08)	**Project presentations**
	Final's week (Project presentations and project report)