

**TAM 416 / Aero 452 Introduction to Nonlinear Dynamics and Vibrations**  
**MW 1-2:50pm 401C1 Engineering Hall and on-line, Fall 2024**  
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**Department of Mechanical Science and Engineering**  
**Canvas course site: <https://canvas.illinois.edu/courses/50087>**

## **Syllabus**

We start with an introduction to the basic concepts and methods of the qualitative and quantitative theory of Nonlinear Dynamics and Vibrations. After discussing theorems on the existence and uniqueness of solutions for some general classes of nonlinear oscillators described by ordinary differential equations, we proceed to a description of the dynamics in phase space. We introduce the definitions of dynamical flow, equilibrium points, and periodic, quasiperiodic and chaotic orbits. We then examine basic asymptotic methods for analyzing the free and forced responses of single- and multi-degree-of-freedom nonlinear oscillators, including the methods of Lindstedt-Poincaré, averaging, and multiple-scales. In addition, we discuss the concept of slow-fast partitions of the dynamics and the method of complexification-averaging (CX-A) for strongly nonlinear (even non-linearizable) systems. We continue with a systematic study of forced (fundamental, subharmonic and superharmonic) resonances, internal resonances and combination resonances that encountered in broad classes of dynamical systems, together with a discussion of linearized stability analysis. This leads to a detailed discussion and study of linearized stability, Floquet theory, the theory of linear parametrically excited systems, and the notion of parametric resonance. Examples are given with the Mathieu, Hill and Mathieu-Hill oscillators, leading to the derivation of stability diagrams in their corresponding parameter planes. We then provide a detailed discussion of time-discrete dynamical systems (nonlinear maps) and show how this class of dynamical systems can be used to study the global dynamics of low dimensional nonlinear oscillators. The concept of Poincaré map is introduced in detail, and systematic techniques for its numerical computation are discussed. We then proceed to an introduction of the concepts of nonlinear normal mode, nonlinear localization and targeted energy transfers in systems of coupled nonlinear oscillators. We conclude with a brief introduction to nonlinear vibrations of elastic continua with applications to nonlinear beam theory. Numerous applications of nonlinear dynamics and vibrations in mechanics and engineering are presented throughout the course.

## **Course organization**

This course has an on-line component, so both on-campus and off-campus students are registered. Attendance of the lectures by the on-campus students is mandatory, while the off-campus students should visit the canvas course website and, <https://mediaspace.illinois.edu/channel/channelid/351520402> for course material. You should be able to access the course web site with your Network ID and password. The IT contact person for the on-line component of the course is:

Sara Smith (she/her)  
Office Support Associate  
Office of Online & Professional Engineering Programs  
Grainger College of Engineering

University of Illinois  
217-244-3611  
[ssmith19@illinois.edu](mailto:ssmith19@illinois.edu)

Class material, including announcements, lecture notes, homework (HW) postings and solutions to the HWs, will be uploaded to the course canvas site. Please make sure to visit that site often to remain updated on course developments.

There will be a regular series of HW assignments which will be posted on Canvas. For on-campus students, HW solutions are due in class on the due dates specified. Off-campus students should submit their HW solutions in pdf form to the TA with cc to me, on the specified due dates. Due to logistical constraints and reasons of fairness to your fellow students, late HW solutions won't be accepted and won't be given any credit, unless there is a documented serious reason (e.g., medical). You are encouraged to collaborate and cooperate with your peers on the HW assignments; however, you should hand in your own original HW. To get full credit, your HW solutions need to show great attention to detail, show all steps of your derivations and be written in a systematic neat, organized and clear way; moreover, if the tasks involve numerical computations you'll need to include a list of your codes, and the computer-generated plots need to show attention to detail, e.g., there should be labels on the axes, correct depiction of vectors, etc.

Office hours of the instructor and the TA will be announced once the semester starts after consultation with the students.

### **Grade breakdown**

Homework: 50% of total grade

Final Exam: 50% of total grade

### **Recommended (not required) references**

Nayfeh, A.H., and Mook, D., 1985, *Nonlinear Oscillations*, Wiley Interscience, New York

Rand, R.H., 2001, *Lecture Notes on Nonlinear Vibrations*, <http://hdl.handle.net/1813/28989>

Verhulst, F., 1990, *Nonlinear Differential Equations and Dynamical Systems*, Springer Verlag

### **Prerequisites**

Basic theory of ordinary differential equations and linear algebra, basic linear dynamics and vibrations

### **Anti-racism and inclusivity statement**

There is the need to raise awareness of the ongoing threat of bias and racism and of the need to take personal responsibility in creating an inclusive learning environment.

The Grainger College of Engineering is committed to the creation of an anti-racist, inclusive community that welcomes diversity along a number of dimensions, including, but not limited to, race, ethnicity and national origins, gender and gender identity, sexuality, disability status, class, age, or religious beliefs. The College recognizes that we are learning together in the midst of the Black Lives Matter movement, that Black, Hispanic, and Indigenous voices and contributions have largely either been excluded from, or not recognized in, science and engineering, and that both overt racism and micro-aggressions threaten the well-being of our students and our university community.

The effectiveness of this course is dependent upon each of us to create a safe and encouraging learning environment that allows for the open exchange of ideas while also ensuring equitable opportunities and respect for all of us. Everyone is expected to help establish and maintain an environment where students, staff, and faculty can contribute without fear of personal ridicule, or intolerant or offensive language. If you witness or experience racism, discrimination, micro-aggressions, or other offensive behavior, you are encouraged to bring this to the attention of the course director if you feel comfortable. You can also report these behaviors to the Bias Assessment and Response Team. Based on your report, BART members will follow up and reach out to students to make sure they have the support they need to be healthy and safe. If the reported behavior also violates university policy, staff in the Office for Student Conflict Resolution may respond as well and will take appropriate action.

### **Sexual misconduct reporting obligation**

The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University's Title IX Office. In turn, an individual with the Title IX Office will provide information about rights and options, including accommodations, support services, the campus disciplinary process, and law enforcement options.

A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality, can be found here: [wecare.illinois.edu/resources/students/#confidential](http://wecare.illinois.edu/resources/students/#confidential).

Other information about resources and reporting is available here: [wecare.illinois.edu](http://wecare.illinois.edu).

### **Academic integrity**

The University of Illinois at Urbana-Champaign Student Code should also be considered as a part of this syllabus. Students should pay particular attention to Article 1, Part 4: Academic Integrity. Read the Code at the following URL: <http://studentcode.illinois.edu/>.

Academic dishonesty may result in a failing grade. Every student is expected to review and abide by the Academic Integrity Policy: <https://studentcode.illinois.edu/article1/part4/1-401/>. Ignorance is not an excuse for any academic dishonesty. It is your responsibility to read this policy to avoid any misunderstanding. Do not hesitate to ask the instructor(s) if you are ever in doubt about what constitutes plagiarism, cheating, or any other breach of academic integrity.

### **Religious observances**

Illinois law requires the University to reasonably accommodate its students' religious beliefs, observances, and practices in regard to admissions, class attendance, and the scheduling of examinations and work requirements. You should examine this syllabus at the beginning of the semester for potential conflicts between course deadlines and any of your religious observances. If a conflict exists, you should notify your instructor of the conflict and follow the procedure at <https://odos.illinois.edu/community-of-care/resources/students/religious-observances/> to request appropriate accommodations. This should be done in the first two weeks of classes.

### **Disability-related accommodations**

To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services

(DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail [disability@illinois.edu](mailto:disability@illinois.edu) or go to <https://www.disability.illinois.edu>. If you are concerned you have a disability-related condition that is impacting your academic progress, there are academic screening appointments available that can help diagnosis a previously undiagnosed disability. You may access these by visiting the DRES website and selecting “Request an Academic Screening” at the bottom of the page.

### **Family educational rights and privacy act (FERPA)**

Any student who has suppressed their directory information pursuant to Family Educational Rights and Privacy Act (FERPA) should self-identify to the instructor to ensure protection of the privacy of their attendance in this course. See <https://registrar.illinois.edu/academic-records/ferpa/> for more information on FERPA.

### **Note regarding public safety**

Emergencies can happen anywhere and at any time, so it’s important that we take a minute to prepare for a situation in which our safety could depend on our ability to react quickly. Take a moment to learn the different ways to leave this building. If there’s ever a fire alarm or something like that, you’ll know how to get out and you’ll be able to help others get out. Next, figure out the best place to go in case of severe weather – we’ll need to go to a low-level in the middle of the building, away from windows. And finally, if there’s ever someone trying to hurt us, our best option is to run out of the building. If we cannot do that safely, we’ll want to hide somewhere we can’t be seen, and we’ll have to lock or barricade the door if possible and be as quiet as we can. We will not leave that safe area until we get an Illini-Alert confirming that it’s safe to do so. If we can’t run or hide, we’ll fight back with whatever we can get our hands on. If you want to better prepare yourself for any of these situations, visit [police.illinois.edu/safe](http://police.illinois.edu/safe). Remember you can sign up for emergency text messages at [emergency.illinois.edu](http://emergency.illinois.edu). Also, please refer to the syllabus attachment next page.

## Run > Hide > Fight

Emergencies can happen anywhere and at any time. It is important that we take a minute to prepare for a situation in which our safety or even our lives could depend on our ability to react quickly. When we're faced with almost any kind of emergency – like severe weather or if someone is trying to hurt you – we have three options: Run, hide or fight.



### Run

**Leaving the area quickly is the best option if it is safe to do so.**

- ▶ Take time now to learn the different ways to leave your building.
- ▶ Leave personal items behind.
- ▶ Assist those who need help, but consider whether doing so puts yourself at risk.
- ▶ Alert authorities of the emergency when it is safe to do so.



### Hide

**When you can't or don't want to run, take shelter indoors.**

- ▶ Take time now to learn different ways to seek shelter in your building.
- ▶ If severe weather is imminent, go to the nearest indoor storm refuge area.
- ▶ If someone is trying to hurt you and you can't evacuate, get to a place where you can't be seen, lock or barricade your area if possible, silence your phone, don't make any noise and don't come out until you receive an Illini-Alert indicating it is safe to do so.



### Fight

**As a last resort, you may need to fight to increase your chances of survival.**

- ▶ Think about what kind of common items are in your area which you can use to defend yourself.
- ▶ Team up with others to fight if the situation allows.
- ▶ Mentally prepare yourself – you may be in a fight for your life.

Please be aware of people with disabilities who may need additional assistance in emergency situations.

## Other resources

- ▶ [police.illinois.edu/safe](https://police.illinois.edu/safe) for more information on how to prepare for emergencies, including how to run, hide or fight and building floor plans that can show you safe areas.
- ▶ [emergency.illinois.edu](https://emergency.illinois.edu) to sign up for Illini-Alert text messages.
- ▶ **Follow the University of Illinois Police Department** on Twitter and Facebook to get regular updates about campus safety.