TAM 598 – Soft Solids

Instructor: Prof. Sascha Hilgenfeldt Contact: 3032C LuMEB, <u>sascha@illinois.edu</u> (please include "TAM598" in subject line)

Lectures: Tu/Th 1:00pm-2:50pm, 2051 LuMEB (4 credit hours) Office Hours: TBD (by appointment is possible) Course Website: hosted on Canvas, <u>https://canvas.illinois.edu/courses/45003</u>



Course Overview: This course focuses on the physical principles and quantitative methods of describing the mechanical behavior of soft materials, particularly those of biological relevance (polymer networks, lipids, cell membranes). It has strong mathematics components, including an introduction to both variational calculus and differential geometry. Students will learn rigorous treatment of biological/soft matter topics relevant in current research, such as network and membrane mechanics, adhesion, and entropic effects. Practical exercises in numerical simulation are integrated into the course.

Skills students will acquire: You will learn how to

- 1. Rigorously apply continuum mechanics and thermodynamics to soft, highly deformable materials encountered in biology.
- 2. Explain mechanical stability and structure of fibers, networks, and membranes
- 3. Use statistical mechanics and entropic concepts to understand and derive mechanical properties of soft materials
- 4. Use differential geometry and calculus of variations to determine equilibrium shapes of vesicles and cells
- 5. Apply computational tools to explore numerical solutions to soft-matter problems

Prerequisites: TAM 541 or CHBE 521 or PHYS 508 for a foundation of mathematical methods is required. Consent of instructor may substitute this requirement. A thorough understanding of thermodynamics and partial differential equations is very helpful. Intermediate solid mechanics is helpful.

A <u>textbook</u> accompanying parts of this course (but not replacing it) is:

David Boal, *Mechanics of the Cell*, Cambridge University Press (2002). Other sources of information are indicated on the literature list and will be drawn upon as needed

Evaluation: Assessment of course outcomes will be done through problem sets, computational exercises, interactive discussion of problems, and a final project. Emphasis will be put on the students' ability to present solutions to the class and on open-ended problems. The final project consists of a student presentation and a written report, both assessing the ability of the students to digest topical scientific work using knowledge from the course and to put it into a broader perspective beyond the course material.

Grading: the tentative weighting of assessments will be

- Homework: 20%
- Interactive discussion: 20%
- Computational exercises: 20%
- Final Project: 40%





Course Outline:

- 1. *Continuum mechanics* Stress, strain, moduli, 3-D and 2-D elastic media, constitutive relations, finite deformations, mesoscale structures and their importance
- 2. Thermodynamics of continua

Energy functionals and deformation; Legendre transformations; entropy and configuration space; statistical mechanics; microstates and macrostates

3. One-dimensional structures

Energetic and entropic springs, linear and nonlinear models (FENE); rubber, neo-Hookean model; biopolymers (spectrin, DNA, cytoskeleton); spring networks; statistics, role of fluctuations, persistence length; FJC, WLC, self-avoidance

Computational exercises: network coordination and stability, soft modes, collapse, prestress

- 4. *Differential Geometry of surfaces* Parametrizations, curvatures, Fundamental Forms, metric tensors, Gauss-Bonnet theorem; geodesic curvature; Monge gauge
- 5. *Two-dimensional structures: interfaces and membranes* Pure interfaces, bubbles, surfactants, lipids, monolayers, Wilhelmy plate, surface pressure; bilayers and membranes; vesicles and liposomes; osmotic pressure plate and membrane bending, Helfrich functional, line tension, hole formation and closure

Computational exercises: interfaces under tension, bubbles, foams

- 6. *Calculus of Variations* Functional derivative; multi-variable variational calculus in continuum media; Lagrange multipliers, constraints; problems of the first and second kind
- 7. *Shells, Membranes, and Vesicle shapes* Buckling, Fourier modes, thermal fluctuations and their spectrum, mode analysis, area difference elasticity model; phase diagrams

Computational exercises: Membrane closure, vesicle shapes, variation of reduced volume

Contact, Adhesion, and Motion of soft surfaces
 Cell-substrate and cell-cell adhesion; cadherins and integrins; contact forces and tensions;
 Hertzian contact; mechanical description of tissues as soft continua and as viscoelastic materials;
 tissue segregation (differential adhesion); homeostatic pressure; spreading dynamics of cells and cell clusters







