

# MICROHYDRODYNAMICS OF SOFT MATERIALS

## MSE 598X FALL 2023

### Instructor:

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TA office hours: TBA

### Course Website:

For course material: <https://canvas.illinois.edu/>

### Text:

*Microhydrodynamics, Brownian Motion, and Complex Fluids* by M. D. Graham (2018), and  
*Course Notes*, by C. M. Schroeder

### Lectures:

Tuesdays and Thursdays, 9:30am-10:50am  
2045 Sidney Lu Mechanical Engineering Building  
We will have approximately 26 lectures 80-minutes in length.

### Problem Sets & Exams:

There will be approximately 5 problems sets. Group work is encouraged. In addition, there will be a group project and a take home midterm exam and final exam.

### Grading:

Homework	25%
Group project	20%
Midterm exam (take home)	25%
Final exam (take home)	30%

**Course Credits:** 4 credits (grad-level); contact hours: 3 hours of lecture per week, plus an additional weekly problem session as needed

**Prerequisites:** MSE 401 (or equivalent); MSE 402 (or equivalent, can be taken simultaneously); Math 285 (or equivalent)

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## Course Description:

This course will provide an introduction to the dynamics of complex fluids at small scales affecting the structure and properties of soft materials such as colloidal particles, polymers, and vesicles. This course will also explore the physical underpinnings of Brownian motion to gain a quantitative understanding of the non-equilibrium dynamics of colloidal suspensions and polymer solutions.

*Learning outcomes:* A wide array of problems in materials science and engineering rely on the processing of colloidal suspensions, polymer solutions, surfactants, and self-assembled materials. It is of paramount importance to quantitatively understand the non-equilibrium properties of these materials. Emphasis is placed on the dynamics under *non-equilibrium* conditions such as fluid flow. These concepts can be broadly applied to a wide range of materials with nontrivial microstructures such as polymer solutions, swimming bacteria, and fiber suspensions that govern the cytoskeleton.

By the end of the course, students will gain a complete understanding of: Newtonian fluid dynamics, including continuum mechanics, kinematics, and mass and momentum balances; fundamental solutions to the Stokes equations, including how these relations describe the viscosity of colloidal suspensions; hydrodynamic interactions between particles in a fluid and hydrodynamic screening in suspensions, including quantitative descriptions of these behaviors; disturbance flows resulting from particle motion in a fluid, including self-propelled microorganisms and/or active colloidal particles; dynamics of non-spherical particles such as rigid rods and anisotropic particles; stress in non-Brownian suspensions of spheres and fibers, developed from a first principle basis with relation to the stresslet; the phenomenon of Brownian motion, including the classical Langevin equation and the relevance of thermal fluctuations; Brownian diffusion for suspended particles, including an introduction to elementary Brownian dynamics simulations and the mathematical foundations of stochastic processes, including the Wiener process, stochastic differential equations, and random walks; development of stress in a Brownian suspension of rods.

Additional learning outcomes will include an understanding of the non-equilibrium dynamics of polymer solutions, including stress in a dilute solution of polymers and development of appropriate constitutive equations (stress-strain rate relations). Advanced topics may include: the non-equilibrium dynamics of semi-dilute unentangled and entangled polymer solutions, the basic aspects of entangled solutions and melts, and stress in branched polymer solutions and melts.

## Course topics (tentative):

1. Kinematics and balance equations
  - Index (Einstein) notation
  - Kinematics, deformation tensors, introduction to convected derivatives
  - Conservation equations
  - General properties of Stokes flow and Lorentz reciprocal theorems
2. Fundamental solution to the Stokes equations & point particles

- Free space Green's function, Stokeslet, point source, point source dipole
  - Stresslet, rotlet
  - Multipole expansions
  - Average stress in a suspension
  - Mobility in a system of particles
3. Flow of finite sized particles: dilute suspensions
- Harmonic function expansions
  - Spheres in translation, rotation, and in a general linear flow; viscosity in a dilute suspension
  - Faxen's laws
  - Microscale swimmers
  - Non-spherical particles & Jeffery orbits
  - Slender body theory
4. Fiber suspensions and anisotropic particles (non-Brownian)
- Application of slender body theory
  - Kinetic theory for rod orientation distributions in flow
  - Stress in a dilute suspension of rigid rods
5. Brownian motion
- Brownian motion of a particle in a fluid: the Langevin equation
  - Correlations of velocity, fluctuating force, and fluctuation-dissipation theorem
  - Diffusion & Taylor-Green-Kubo formula
  - Basic Brownian dynamics
6. Stochastic differential equations & Brownian rod suspensions
- Diffusion equation and the Wiener process
  - Stochastic calculus - overview and basics
  - Time evolution of probability & Fokker-Planck equation
  - Revisiting the Langevin equation
  - Kinetic theory for Brownian rod suspensions
7. Polymer solutions
- Polymer models & equilibrium chain statistics
  - Equilibrium & internal degrees of freedom
  - Kinetic theory for a Hookean dumbbell
  - Polymer contribution to stress tensor
  - Finite extensibility & shear thinning
  - Bead-spring polymer models
8. Viscoelastic flow phenomena (time permitting)
- Review of linear viscoelasticity
  - Generalized Stokes-Einstein relation (GSER) and microrheology
  - Nonlinear viscoelasticity: shear and extensional flows

### Course Outline (tentative)

Lec	Topic	Reading*
Lec 1	Course intro; complex fluids, constitutive equations; index (Einstein) notation; review of tensors	L §1; P §3; G §A.1
Lec 2	Differential and integral calculus with vectors & tensors; kinematics of fluids	G §1.1; P §3
Lec 3	Shear, extensional, and mixed flow; Lagrangian and Eulerian specification; deformation tensors	G §1.1
Lec 4	Conservation of mass; conservation of momentum; statics; dynamic stress & deviatoric stress	G §1.2
Lec 5	Constitutive equations & Navier-Stokes equation	G §1.2
Lec 6	Creeping flow; Stokeslet; fundamental solutions to the Stokes equation	G §1.3, 2.1
Lec 7	Point source & higher order singularities; sources/sinks; point source dipoles	G §2.2
Lec 8	Point force singularities; force dipoles; stresslet & rotlet; multipole expansions	G §2.3, 2.4
Lec 9	Three-dimensional creeping flows: beyond point particles; vector harmonics; forces and torques on particles	G §3.1, 3.2
Lec 10	Flow past solid boundaries; integral force & torque theorems	G §1.2.3
Lec 11	Harmonic function expansion & 3D flows, finite sized-particles	G §3.3
Lec 12	Sample problems: flow-directed assembly of particles; sphere in linear flow; model microscale swimmer	G §3.4
Lec 13	Stress in a dilute suspension of non-Brownian particles; Einstein's viscosity for a dilute suspension of spheres	G §3.3.3
	Midterm exam	
Lec 14	Mobility of a system of point particles; reciprocal theorem	G §2.5, 1.3.3
Lec 15	Faxen's law & mobility; mobility of a system of finite-sized particles; symmetry relations for resistivity tensors	G §3.5, 3.6
Lec 16	Non-spherical rigid particles; spheroid in general linear flow	G §3.7, 3.8
Lec 17	Slender body theory; stress in a dilute suspension of anisotropic particles	G §3.9
Lec 18	Stress in non-Brownian suspension of rods; steady extensional stress of rigid rod suspension	
Lec 19	Fokker-Planck equation & general flows; simple shear flow	
Lec 20	Brownian motion & thermal fluctuations; Langevin equation; fluctuation-dissipation theorem	G §6.1, 6.2
Lec 21	Diffusion & velocity autocorrelation functions; Green-Kubo relations	G §6.5, 6.6
Lec 22	Brownian suspension of rigid rods	G §8.7
Lec 23	Dynamics of Brownian rod suspension; Brownian relaxation; Brownian stress	G §7.7, 8.7
Lec 24	Dynamics of flexible polymers in solution: FJC, FRC, HRC; persistence length; radius-of-gyration; entropic restoring force	G §8.1, 8.2
Lec 25	Hookean dumbbell model for polymer chain; kinetic theory; dynamics of flexible polymers; effective stress in solution of Hookean dumbbells	G §8.5, 8.6
Lec 26	General entropic spring forces; dynamics of non-linear dumbbells; Oldroyd-B model; time-dependent stress in uniaxial flow; upper convected Maxwell derivative	
Lec 27	Finitely extensible polymers; dynamics and stress for non-linear spring model; Giesekus expression for stress; Warner spring; shear thinning	

\*Refer to page 6 of the syllabus for textbook acronyms.

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## Lecture Dates

### August

<b>Tuesday</b>	<b>Thursday</b>
Aug 22 / Lec 1	Aug 24 / Lec 2
Aug 29 / Lec 3	Aug 31 / Lec 4

### September

<b>Tuesday</b>	<b>Thursday</b>
Sep 5 / Lec 5	Sep 7 / No class (travel)
Sep 12 / Lec 6	Sep 14 / Lec 7
Sep 19 / Lec 8	Sep 21 / Lec 9
Sep 26 / Lec 10	Sep 28 / Lec 11

### October

<b>Tuesday</b>	<b>Thursday</b>
Oct 3 / Lec 12	Oct 5 / Lec 13
Oct 10 / Lec 14	Oct 12 / Take-home Midterm (No class)
Oct 17 / Lec 15	Oct 19 / Lec 16
Oct 24 / Lec 17	Oct 26 / Lec 18

### November

<b>Tuesday</b>	<b>Thursday</b>
Oct 31 / Lec 19	Nov 2 / Lec 20
Nov 7 / No class (travel)	Nov 9 / Lec 21
Nov 14 / Lec 22	Nov 16 / Lec 23
Nov 21 / No class (Fall Break)	Nov 23 / No class (Fall Break)
Nov 28 / Lec 24	Nov 30 / Lec 25

### December

<b>Tuesday</b>	<b>Thursday</b>
Dec 5 / Lec 26	

**Midterm Exam:** Take-home - Oct 11-13

**Final Exam:** Take-home - December 6-8

**Projects - written:** Due December 5

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## Textbooks that I have found useful:

### Complex fluids:

- *Dynamics of Polymeric Liquids: Fluid Mechanics*, R. B. Bird, R. C. Armstrong, O. Hassager, Volume 1, 2nd edition, Wiley (1987).
- *Dynamics of Polymeric Liquids: Kinetic Theory*, R. B. Bird, C. F. Curtiss, R. C. Armstrong, O. Hassager, Volume 2, 2nd edition, Wiley (1987).
- *The Structure and Rheology of Complex Fluids*, R. G. Larson, Oxford Univ. Press (1999).  
**Acronym: L**
- *Polymer Physics*, M. Rubinstein and R. H. Colby, Oxford Univ. Press (2003).
- *The Theory of Polymer Dynamics*, M. Doi and S. F. Edwards, Clarendon Press (1986).
- *Colloidal Dispersions*, W. B. Russel, D. A. Saville, W. R. Schowalter, Cambridge Univ. Press (1989).

### Fluid mechanics:

- *Microhydrodynamics: Principles and Selected Applications*, S. Kim and S. J. Karrila, Butterworth (1991).
- *Analysis of Transport Phenomena*, W. Deen, Oxford University Press (1998).
- *Advanced Transport Phenomena*, L. G. Leal, Cambridge Univ. Press (2007).
- *An Introduction to Fluid Dynamics*, G. K. Batchelor, Cambridge Univ. Press (1967).
- *Incompressible Flow*, R. L. Panton, 3rd edition, Wiley (2005). **Acronym: P**
- *Low Reynolds Number Hydrodynamics*, J. Happel and H. Brenner, Martinus Nijhoff (1983).

### Mathematics:

- *Advanced Engineering Mathematics*, M. Greenberg, 2nd edition, Prentice Hall (1998).
- *Elementary Differential Equations*, Boyce and DiPrima, 9th edition, Wylie (2008).
- *Introduction to Tensor Analysis*, H. D. Block (1978).
- *Vectors, Tensors, and the Basic Equations of Fluid Mechanics*, R. Aris, Dover Publications (1962).

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### Course Policies:

- **Submitting assignments.** All assignments are due on the assignment due date (no later than 11:59pm) as listed in the syllabus. Please submit your coursework either in class or on the course website by following the instructions provided in class.
- **Late assignments.** All late assignments will be subject to a 25% per day penalty, unless the absence is officially excused. Missed exams without a valid excuse will result in a zero on the exam.
- **Academic misconduct** “The University of Illinois at Urbana-Champaign expects its faculty, staff, students and guests to conduct themselves in accordance with the community values of civility, respect, and honesty; to maintain the highest level of integrity and exercise critical judgment in all dealings, decisions and encounters; and to maintain and strengthen the public’s trust and confidence in our institution.” This and other text regarding academic integrity expectations can be found in the University Student Code found at: <http://admin.illinois.edu/policy/code/index.html>, specifically, here: <https://studentcode.illinois.edu/article1/part4/1-402/>

Academic misconduct (cheating, plagiarism, etc.), as a form of fraud, undermines the public trust, both in the institution and in the degree. When you sign your name to work, you are stating that the work is yours, you created it or contributed to it, and you comprehend everything in it. Instructors are required to report all suspected infractions of academic integrity in the online FAIR system that guides both the instructor and the student through the different phases of the process exactly as stated in the Student Code, including any appeals regarding the finding and/or the sanction. You are encouraged to read and carefully consider the University of Illinois at Urbana-Champaign Student Code as a part of this syllabus. Ignorance is not an excuse for any academic dishonesty. It is your responsibility to read this policy to avoid any misunderstanding. Do not hesitate to ask the instructor(s) if you are ever in doubt about what constitutes plagiarism, cheating, or any other breach of academic integrity.

- **Group work versus individual work** For all individual assignments, the work that you turn in must be your own work. Working in groups to discuss overall approaches to solving problems is fine (and encouraged), but you need to turn in your own work for individual assignments. Penalties are associated with any forms of cheating. University and departmental policies will be followed in the case of any suspected cheating incidents.
- **Copyright course materials.** All materials for this course are considered copyright of the University of Illinois at Urbana-Champaign. It is wholly unacceptable for students to post course materials (homeworks, quizzes, exams, solutions, lecture notes, etc.) in public places, including unauthorized websites, for sale or otherwise. Any act of making course materials available on the WWW, or in any other format, is considered copyright violation.

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**Sexual Misconduct Reporting Obligations:** The University of Illinois is committed to combating sexual misconduct. Faculty and staff members are required to report any instances of sexual misconduct to the University's Title IX Office. In turn, an individual with the Title IX Office will provide information about rights and options, including accommodations, support services, the campus disciplinary process, and law enforcement options. A list of the designated University employees who, as counselors, confidential advisors, and medical professionals, do not have this reporting responsibility and can maintain confidentiality, can be found here: [wecare.illinois.edu/resources/students/#confidential](http://wecare.illinois.edu/resources/students/#confidential). Other information about resources and reporting is available here: [wecare.illinois.edu](http://wecare.illinois.edu).

**Religious Observances:** Illinois law requires the University to reasonably accommodate its students' religious beliefs, observances, and practices in regard to admissions, class attendance, and the scheduling of examinations and work requirements. You should examine this syllabus at the beginning of the semester for potential conflicts between course deadlines and any of your religious observances. If a conflict exists, you should notify your instructor of the conflict and follow the procedure at <https://odos.illinois.edu/community-of-care/resources/students/religious-observances/> to request appropriate accommodations. This should be done in the first two weeks of classes.

**Disability-Related Accommodations:** To obtain disability-related academic adjustments and/or auxiliary aids, students with disabilities must contact the course instructor and the Disability Resources and Educational Services (DRES) as soon as possible. To contact DRES, you may visit 1207 S. Oak St., Champaign, call 333-4603, e-mail [disability@illinois.edu](mailto:disability@illinois.edu) or go to: <https://www.disability.illinois.edu>

**Family Educational Rights and Privacy Act (FERPA):** Any student who has suppressed their directory information pursuant to Family Educational Rights and Privacy Act (FERPA) should self-identify to the instructor to ensure protection of the privacy of their attendance in this course. See <https://registrar.illinois.edu/academic-records/ferpa/> for more information on FERPA.

### **Statement on Anti-Racism:**

The Grainger College of Engineering is committed to the creation of an anti-racist, inclusive community that welcomes diversity along a number of dimensions, including, but not limited to, race, ethnicity and national origins, gender and gender identity, sexuality, disability status, class, age, or religious beliefs. The College recognizes that we are learning together in the midst of the Black Lives Matter movement, that Black, Hispanic, and Indigenous voices and contributions have largely either been excluded from, or not recognized in, science and engineering, and that both overt racism and micro-aggressions threaten the well-being of our students and our university community. The effectiveness of this course is dependent upon each of us to create a safe and encouraging learning environment that allows for the open exchange of ideas while also ensuring equitable opportunities and respect for all of us. Everyone is expected to help establish and maintain an environment



where students, staff, and faculty can contribute without fear of personal ridicule, or intolerant or offensive language. If you witness or experience racism, discrimination, micro-aggressions, or other offensive behavior, you are encouraged to bring this to the attention of the course director if you feel comfortable. You can also report these behaviors to the Bias Assessment and Response Team (BART) (<https://bart.illinois.edu/>). Based on your report, BART members will follow up and reach out to students to make sure they have the support they need to be healthy and safe. If the reported behavior also violates university policy, staff in the Office for Student Conflict Resolution may respond as well and will take appropriate action.

**Dealing with Stress and Personal Issues:**

Counseling services are available to all of our students here on campus. College can be stressful for a variety of reasons. The Department believes your mentalhealth is as important as your physical health and intellectual growth. If you are feeling overwhelmed, depressed, or anxious, there are many resources on campus to assist you. The Counseling Center offers same-day first time appointments, time-limited counseling, long-term group therapy, and several skill development workshops. Please call them at 217-333-3704 to make an appointment, or visit their website for more info:

<http://www.counselingcenter.illinois.edu>

If you are experiencing a mental health crisis and feel you are in immediate danger, please call 911. The Champaign County Crisis Line (217-359-4141) is also available 24 hours a day, 7 days a week, 365 days a year.

## Run > Hide > Fight

Emergencies can happen anywhere and at any time. It is important that we take a minute to prepare for a situation in which our safety or even our lives could depend on our ability to react quickly. When we're faced with almost any kind of emergency – like severe weather or if someone is trying to hurt you – we have three options: Run, hide or fight.



### Run

**Leaving the area quickly is the best option if it is safe to do so.**

- ▶ Take time now to learn the different ways to leave your building.
- ▶ Leave personal items behind.
- ▶ Assist those who need help, but consider whether doing so puts yourself at risk.
- ▶ Alert authorities of the emergency when it is safe to do so.



### Hide

**When you can't or don't want to run, take shelter indoors.**

- ▶ Take time now to learn different ways to seek shelter in your building.
- ▶ If severe weather is imminent, go to the nearest indoor storm refuge area.
- ▶ If someone is trying to hurt you and you can't evacuate, get to a place where you can't be seen, lock or barricade your area if possible, silence your phone, don't make any noise and don't come out until you receive an Illini-Alert indicating it is safe to do so.



### Fight

**As a last resort, you may need to fight to increase your chances of survival.**

- ▶ Think about what kind of common items are in your area which you can use to defend yourself.
- ▶ Team up with others to fight if the situation allows.
- ▶ Mentally prepare yourself – you may be in a fight for your life.

Please be aware of people with disabilities who may need additional assistance in emergency situations.

## Other resources

- ▶ [police.illinois.edu/safe](https://police.illinois.edu/safe) for more information on how to prepare for emergencies, including how to run, hide or fight and building floor plans that can show you safe areas.
- ▶ [emergency.illinois.edu](https://emergency.illinois.edu) to sign up for Illini-Alert text messages.
- ▶ **Follow the University of Illinois Police Department** on Twitter and Facebook to get regular updates about campus safety.