IE 518: Queueing Systems

Instructor: Alexander Stolyar

Contact Information   **Course Schedule**

Office: 156 CSL   Tue-Th, 3:30-4:50
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Office hours (tentative, will adjust if necessary)

Tue 5-6 and Th 5-6, and by appointment; 156 CSL

Course Description

Queueing theory is a powerful tool for analysis and design of a wide range of engineering systems. Modern applications include: inventory and manufacturing systems; sharing economy platforms; transportation; service systems; telecommunication, informaton and computing systems; network clouds; peer-to-peer networks; and others. This course is an introduction to queueing systems and their applications in engineering. Topics include both classical single-stage models and queueing networks. Students will learn how to apply key ideas, methods and tools of queueing theory, such as: Markov processes, embedded Markov chains, PASTA property, reversibility, product-form stationary distributions, stochasic stability, asymptotic analysis.

Prerequisites

IE410 or an equivalent graduate stochastic processes course

Learning outcomes

Learn foundations of queueing theory: basic models, key ideas and methods.

Understand how to apply queueing theory to model and analyze engineering systems.

Develop background and skills, which will allow students to subsequently study other and/or more advanced topics in queueing theory.

Text

L. Kleinrock, Queueing Systems, Vol. 1, Wiley, 1975.

Optional texts for probability and stochastic processes background

S. M. Ross, Introduction to Probability Models, 11th Ed., Academic Press, 2014.
S. M. Ross, Stochastic Processes, 2nd Ed., Wiley, 1996.
G. Lawler. Introduction to Stochastic Processes, 2nd Ed., Chapman & Hall, 2006.

Course outline (tentative)

1. Introduction (1 week)
   a. Review of the probability and stochastic processes prerequisites
   b. Laplace transforms and generating functions
   c. Queueing notation and basics
2. Single-stage Markov systems (2 weeks)
   a. Birth-death systems in stationary regime
   b. Relation between distributions at arrivals and time-averages; PASTA (Poisson Arrivals See Time Averages)
   c. Systems with phase-type inter-arrival and service time distributions
3. Insensitivity (w.r.t. service time distribution) property for some models (1 week)
   a. Loss model (M/GI/m/m) and generalizations
   b. Last-Come-First-Serve model (M/GI/1-PreemptiveLCFS)
4. M/GI/1 system (2 weeks)
   a. Embedded Markov chain (at departures); Relation between distributions at arrivals, departures and time averages
   b. Pollacheck-Khinchine formula for the stationary distribution
   c. Tail decay rate of the stationary distribution
   d. Stationary distribution of sojourn time, waiting time, unfinished work
   e. System busy period
5. GI/M/m system (1 week)
   a. Embedded Markov chain (at arrivals) and its stationary distribution
   b. Stationary distribution; Simple solution for GI/M/1
6. GI/GI/m system (1.5 weeks)
   a. GI/GI/1: Lindley recursion for waiting time; Workload and idle time dynamics
   b. Heavy-traffic diffusion limit / approximation
7. Queueing networks (2 weeks)
   a. Tandem network of M/M/m: Output flow in steady-state; Product-form stationary distribution
   b. Jackson network of M/M/m: Product-form stationary distribution
   c. Closed network of M/M/m queues
   d. Multiclass open/closed/mixed product-form networks
8. Queueing networks stability issues (2 weeks)
   a. Possible instability under sub-critical load
   b. Stability proofs via Lyapunov-Foster-type criteria
   c. Fluid limits
9. Possible additional topics

Assignments and grade composition
    Homeworks (approximately 7): 50%, Mid-term exam: 20%, Final exam: 30%

Logistics
- **All assignments are submitted electronically** in canvas, as a scanned PDF file.
- **Mid-term exam** is take-home.
- **Final exam** is take-home, Dec 11 (Monday), 1:30PM – 4:30PM.