IE 598: Topics in Game Theory and Fair Division

Spring 2022

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Course Description

The course will explore various topics at the intersection of economics and computation whose solutions have been deployed to solve a wide-range of real-life settings such as assigning medical residents to hospitals, allocating students to schools, assigning seats in courses, kidney exchange, refugee allocation, assigning public housing, airport traffic management, and so on. The course will cover the topics in foundations of game theory and fair division such as Nash equilibrium, bargaining, mechanism design, fair and efficient allocation of goods/chores, and their computation.

Prerequisites: IE 310 or equivalent; basic knowledge of optimization, probability, and linear algebra; mathematical maturity

References

1. *Game Theory: Analysis of conflict* by Roger Myerson, Harvard Press, 1997.
2. *Twenty Lectures on Algorithmic Game Theory* by Tim Roughgarden, Cambridge, 2016.
3. *A Course in Game Theory* by Osborne and Rubinstein, MIT Press, 1994.
4. *Fair Division: From cake-cutting to dispute resolution* by Steven J Brams and Alan D Taylor, Cambridge University Press, 1996.
5. *Fair division and collective welfare* by Hervé Moulin, MIT press, 2004.
6. *Cake-cutting algorithms: Be fair if you can* by Jack Robertson and William Webb, CRC Press, 1998.

Required Work and Grading Policy

1. 4 Homework assignments (60%)
2. Project (40%)

Tentative Course Outline

- **Week 1-2 (fair division of divisible items):** cut and choose protocol; fairness notions: envy-freeness (EF), proportionality (Prop); efficiency notions: Pareto optimality, (Nash) social welfare; competitive equilibrium
- **Week 3-5 (fair division of indivisible/mixed items):** fairness notions: envy-freeness up to one/any good (EF1/EFX), proportionality up to one good (Prop1), maximin share allocation (MMS); envy-cycle procedure; approximate MMS allocation

- **Week 6-8 (game theory):** Nash equilibrium; minimax theorem; support enumeration algorithm; Sperner's lemma; PPAD-hardness, Lemke-Howson algorithm; correlated equilibrium; bargaining and cooperation: Nash bargaining, core, Shapley value
- **Week 9-11 (Price of Anarchy):** Selfish routing; Potential games; Congestion games; Cost sharing games; best response dynamics; PLS
- **Week 12-13 (Mechanism design):** First price auction, second price (Vickery) auction; Myerson's lemma; VCG mechanism; Mechanism design without money: top trading cycle, kidney exchange
- **Week 14-15:** project presentations

Course Project

A course project can have at most 2 students. The project could be reading a couple of recent research papers, survey of some topic not covered in the class, or on a research problem. The evaluation of project is based on a written report (8-10 pages), class presentation and class feedback. We will have project presentations at the end of the course.