

# IE598: Queueing Systems (Spring 2017)

Instructor: A. Stolyar

## Contact Information:

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## Course Schedule:

Tue-Th 5:00-6:20  
1131 Siebel Center

## Course Description

This course is an introduction to queueing systems and their applications in engineering. Topics include both classical single-stage models and queueing networks. Students will learn how to apply key ideas and methods of queueing theory, such as: embedded Markov chains, PASTA property, reversibility, product-form stationary distributions, stochastic stability, asymptotic analysis.

## Prerequisites

IE410 or an equivalent stochastic processes course.

## Text

L. Kleinrock, Queueing Systems, Vol. 1, Wiley, 1975.

## Optional texts for probability and stochastic processes background

S. M. Ross, Introduction to Probability Models, 11th Ed., Academic Press, 2014.

S. M. Ross, Stochastic Processes, 2nd Ed., Wiley, 1996.

G. Lawler. Introduction to Stochastic Processes, 2nd Ed., Chapman & Hall, 2006.

## Course outline (tentative)

1. Introduction (1 week)
  - a. Review of the probability and stochastic processes prerequisites
  - b. Laplace transforms and generating functions
  - c. Queueing notation and basics
2. Single-stage Markov systems (2 weeks)
  - a. Birth-death systems in stationary regime
  - b. Relation between distributions at arrivals and time-averages; PASTA (Poisson Arrivals See Time Averages)
  - c. Systems with phase-type inter-arrival and service time distributions
3. Insensitivity (w.r.t. service time distribution) property for some models (1 week)
  - a. Loss model (M/GI/m/m) and generalizations
  - b. Last-Come-First-Serve model (M/GI/1-PreemptiveLCFS)
4. M/GI/1 system (2 weeks)
  - a. Embedded Markov chain (at departures); Relation between distributions at arrivals, departures and time averages

- b. Pollacheck-Khinchine formula for the stationary distribution
  - c. Tail decay rate of the stationary distribution
  - d. Stationary distribution of sojourn time, waiting time, unfinished work
  - e. System busy period
5. GI/M/m system (1 week)
- a. Embedded Markov chain (at arrivals) and its stationary distribution
  - b. Stationary distribution; Simple solution for GI/M/1
6. GI/GI/m system (1.5 weeks)
- a. GI/GI/1: Lindley recursion for waiting time; Workload and idle time dynamics
  - b. Heavy-traffic diffusion limit / approximation
7. Queueing networks (2 weeks)
- a. Tandem network of M/M/m: Output flow in steady-state; Product-form stationary distribution
  - b. Jackson network of M/M/m: Product-form stationary distribution
  - c. Closed network of M/M/m queues
  - d. Multiclass open/closed/mixed product-form networks
8. Queueing networks stability issues (2 weeks)
- a. Possible instability under sub-critical load
  - b. Stability proofs via Lyapunov-Foster-type criteria
  - c. Fluid limits

### **Assignments and grade composition**

Homeworks (50%), Mid-term exam (20%), Final exam (30%)